Preliminary evaluation: Problems and questions w.r.t. the "mathematical think-activities" (MTA) in the Dutch exams starting in 2017 (HAVO, pre-college) and 2018 (VWO, pre-university), following eight years of cTWO 2004-2012

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October 31 and November 1 & 16 2016

Abstract

Mathematical insight is not something that you can develop a reasonable expectation about and prepare for. Thus insight should not be part of the final examination in the national curriculum. Insight helps to do exam questions, but those should test issues that one can prepare for, i.e. knowledge and skill. The proper approach to help students develop insight is to improve didactics. Improving didactics by imposing supposed tests on insight in the national exam is a fallacy of composition. Thinking that this can be done in this manner, might cause that the proper approach to improve didactics doesn't get sufficient attention. See http://thomascool.eu/Papers/Math/Index.html

Abbreviations

MTA = "mathematical think-activity" or "-ies"
BAU = business as usual
PE = primary education
SE = secondary education
TE = tertiary education
HAVO = pre-college SE
VWO = pre-university SE
MBO = trade school
RME = realistic mathematics education (ideology)
TME = traditional mathematics education (ideology)
NME = neoclassical mathematics education (scientific)
GC = graphic calculator
cTWO = commissie Toekomst Wiskunde Onderwijs (the commission that created the current programme, from 2015)
KNAW = Royal Dutch Academy of Sciences
NVvW = Nederlandse Vereniging van Wiskundeleraren, Dutch association of mathematics teachers

Summary

Teaching aims are knowledge, skill and attitude. Knowlege and skill can be tested in the final exam of the national curriculum. These are topics that the student can develop a reasonable expectation about and prepare for. The attitude helps for the preparation for the exam but the exam does not test for attitude directly. For mathematical insight the situation is similar to that of attitude. Insight is not something that you can develop a reasonable expectation about and prepare for. Thus insight should not be part of the final examination in the national curriculum. Insight helps to do exam questions, but those should test issues that one can prepare for, i.e. knowledge and skill. The proper approach to help students develop insight is to improve didactics. Improving didactics by imposing supposed tests on insight in the national exam is a fallacy of composition. Thinking that this can be done in this manner, might cause that the proper approach to improve didactics doesn't get sufficient attention.

In Holland 2004-2012, a committee cTWO supplemented by a "Resonansgroep" redesigned the mathematics examination in the Dutch national curriculum. cTWO agreed on the aim of testing mathematical insight, and came up with the name of "mathematical think-activity" (MTA) as the label for education and testing. The regulation on MTA now has legal status. The new programme started in 2015, with exams for HAVO (2 years pre-college) in 2017 and VWO (3 years pre-university) in 2018. However, the implementation is lagging. Teachers and exam designers are still working on the enigma what would be proper MTA, and teachers wondering how to teach it (if it would be really new). See for example the large number of sessions on MTA at the Dutch association of teachers (NVvW) annual convention of 2015.

The only reason why this would not develop into a disaster is likely that teachers and testers behave in "business as usual" (BAU), and give the label "MTA" to the more difficult questions (that perhaps allow more ways of solving). See for example the small number of sessions on MTA at the Dutch association of teachers (NVvW) annual convention of 2016. However, when there would be a higher percentage of such more difficult questions, then weaker students would have to earn their points on a smaller percentage of questions of a normal difficulty.

The cTWO fallacy of composition likely has come about by three kinds of influences. A first possibility is that many of its authors were honestly deluded. A second influence is that if they set the exam criteria, then textbooks would follow, and then teachers would follow. If this has been a deliberate choice by some authors, then it would abuse testing for aims in didactics (a warning by the "Resonansgroep"). The third influence is that cTWO could agree on aims but not on implemention and didactic notions.

In Holland since at least about 2004 there has been a *math war* between the "realistic mathematics education" (RME) (a.k.a. "reform mathematics") and "traditional mathematics education" (TME). These are *ideologies* – comparable to astrology or homeopathy. TME is an ideology of mathematicians who have no qualification for secondary or primary education and its research. RME is an ideology created by a similar mathematician Hans Freudenthal (1905-1990) who also had an interested in education but no scientific training on it and its research. Henk Broer, professor of mathematics and chairman of the mathematics section of the Dutch Academy of Sciences (KNAW), acknowledged in a lecture in 2015 that the math war was still raging in 2015, and he shows himself to be one of the main warriors on behalf of TME. MTA can be seen as a compromise between RME (working in "realistic contexts") and TME (higher standards and proving).

A result of this math war between ideologies is that scientific research on didactics does not get proper attention. This notably concerns the scienctific approach of "neoclassical mathematics education" (NME) that I develop since 2008 in my books "Elegance with Substance" (EWS) (2009, 2015), "Conquest of the Plane" (COTP) (2011), "A child wants nice and no mean numbers" (CWNN) (2015) and "Foundations of Mathematics. A neoclassical approach to infinity" (FMNAI) (2015). See http://thomascool.eu/Papers/Math/Index.html

Contents

Introduction

> NB. This evaluation is preliminary. The topic of the *Dutch national mathematics exam standard* is rather huge. Dozens of people have been working on this since 2004 and I have only picked up some elements over the years. But it is useful now to take some stock. I *might* return to this topic at a later moment and then give a more definitive evaluation (also using comments on this preliminary document). I noticed that some arguments are repeated but this would be in different contexts.

The Dutch secondary education (SE) programme for mathematics has been renewed. The new examinations will be in 2017 for HAVO (2 years pre-college) and in 2018 for VWO (3 years pre-university). [1] [2] [3] The programme e.g. has a return of analytic geometry. [4] Unfortunately computer algebra still is undeveloped. [5]

The renewal has been prepared in 2004-2012 by a commission called *"commissie Toekomst Wiskunde Onderwijs"* (cTWO). [6] [7] A cTWO vision document is "*Rijk aan betekenis*" (2007). [8] Subsequently there were consultations and pilots. The final report was at the end of 2012. The cTWO programme is being implemented for students entering K10 (4th year of secondary education) starting from 2015 onwards.

A key role within the new programme are the *"mathematical think-activities"* (MTA). [9]

Appendix A contains the current regulation. **Appendix B** mentions various sources. **Appendix C** mentions the committee members of cTWO, with footnotes. There was a curious event with a "Resonansgroep" in 2008, see **Appendix D**, so perhaps we should speak about the cTWO-Resonans programme. **Appendices F & G** allow a comparison. **Appendix I** gives a case at the NVvW annual convention in 2015 how the focus on implementing MTA blocks interest for innovation in didactics. **Appendix J** contains some summary comments in Dutch.

Aim and practice of "mathematical think-activities" (MTA)

The **aim of MTA** is:

- Rather than drilling and rote learning, the students are invited to understand mathematics.
- Teachers must pose questions rather than tell how it is.
- MTA wants to stimulate these **aspects** (or competences): (algebraic) modelling, ordening / structuring, analytical thinking and problem solving, manipulating formulas, abstraction, and logical reasoning and proving.

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[1] https://www.nvvw.nl/23132/vakinhoudelijk/examenprogramma-2015
[2] http://www.platformwiskunde.nl/onderwijs_examenprogrammas_vanaf_2015.htm
[3] (i) The difference in duration between VWO and HAVO might be reconsidered. The length of VWO seems to have been determined historically by the consideration that ten·eight year olds are able to move to another town with university. (ii) I think that studying is like working and should come with a salary. This should help the motivation of studying as well. A tax on income earned after graduation would finance the scheme. The government would be the tatonnement master. Given the uncertainties of a future career, this scheme is better than reducing the student to an entrepreneur dealing with a bank. (iii) A problem with highschool might be that students might not be challenged enough. When they enter university, they suddenly have to work.
[4] "Conquest of the Plane" (COTP) (2011) is a primer (teacher training book) for analytic geometry and calculus.
[5] https://boycottholland.wordpress.com/2015/12/08/computer-algebra-is-a-revolution-but-21st-century-skills-q
[6] http://www.fisme.science.uu.nl/ctwo
[7] http://www.fisme.science.uu.nl/wiki/index.php/Ctwo
[8] http://www.fi.uu.nl/ctwo/publicaties/docs/Rijkaanbetekenisweb.pdf
[9] https://www.youtube.com/channel/UCId5vPEUusMLvO8Y8WLcjxw and http://www.slo.nl/agenda/hier-havo and http://www.epsilon-uitgaven.nl/E72.php There is a course on MTA, consisting of 4 sessions of 4 hours, at the cost of EUR 480 per person. http://www.uu.nl/agenda/cursus-wiskundige-denkactiviteiten

Direct comments are:

- A logical order of the aspects would be: (i) Order and structure problems, (ii) make abstractions and (algebraic) models, (iii) apply logic and do proofs. (See: Van Hiele levels, [10] Joop van Dormolen OSAEV, [11] the empirical cycle. [12])
- These aspects are not necessarily the same as insight (understanding), but can be diagnosed as steps that may contribute to insight.
- NB. Perhaps this suggestion of MTA can be seen as an intermediate phase in putting metacognitive insights about "learning how to learn" at center stage. (Lesson 1: be motivated to learn.) See this discussion w.r.t. work by Nijhof et al. (2016). [13]

The **practice of MTA** is different from above laudable aims. At this moment the most detailed discussion that I have been able to find about **grading** such exam questions is in the master's thesis by Hanneke Kodde-Buitenhuis (2016), [14] under supervision of Paul Drijvers. In their scheme the old exams already contained MTA but the new exams should have a higher percentage. This of course depends upon their definition of MTA. It appears to be a notion still looking for implementation.

- A claimed example MTA would be question 3 in VWO B 2014-1, see **Appendix H**, composed from Buitenhuis (2016:83) [15] with the exam question [16] and grading.[17] In the grading there is no explicit reference to modeling, structuring etcetera, however. Buitenhuis actuall tries to identify these elements after the fact.
- When asked [18] what would be the difference between "realistic contexts" and MTA, Van Wijk indicated that context questions have a structured answer (with steps that can be graded with points) while MTA have a diversity in possible answers (with more complex grading). I tend to regard this "rule of thumb" as rather rough and he might reconsider. [19] It is not clear how this relates to Buitenhuis (2016).
  PM. The formal definition of a "context question" is that it contains a variable with a dimension (like kilograms or euros). In that case, the student can use the GC to solve the question.

Confusion and slow implementation of MTA

When we look closer at MTA, we see (which this memo shows):

- confusion and
- a slow implementation: while the exam regulation already has been imposed, test designers and teachers are still working on implementation. This is only "business as usual" (BAU) if we forget about the laudable aims. (Otherwise the exams are invalid and an intellectual fraud, and we should question what CITO is claiming about all of this.)

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[10] Paul Drijvers agrees with me that the list of aspects is a bit problematic, and suggests a summary triangle, but this I find problematic again. Does he really suggest that logical reasoning would be the intersection of modeling and abstraction ? http://www.ru.nl/publish/pages/779511/2016-05-12leonvandenbroeklezingnijmegenwda-drijvers.pdf, sheet 20
[11] http://www.fisme.science.uu.nl/wiki/index.php/OSAEV and recommendable http://www.nieuwarchief.nl/serie5/pdf/naw5-2001-02-4-356.pdf
[12] https://en.wikipedia.org/wiki/Empirical_research#Empirical_cycle
[13] http://thomascool.eu/Papers/Math/Polya/2016-11-15-Implementing-Polya.pdf
[14] http://dspace.library.uu.nl/bitstream/handle/1874/318236/Onderzoek%20naar%20WDA%20in%20pilotexamens_Hanneke%20Kodde_juli%202015.pdf?sequence=2
[15] https://www.examenblad.nl/examen/wiskunde-b-vwo-2/2014
[16] http://static.examenblad.nl/9336114/d/ex2014/vw-1025-f-14-1-o.pdf
[17] http://static.examenblad.nl/9336114/d/ex2014/vw-1025-f-14-1-c.pdf
[18] At the NVvW-SLO conference "Hier HAVO ...", September 28 2016
[19] See also his sheets at http://www.slo.nl/agenda/hier-havo. Sheet 9 gives a triangle by Paul Drijvers, but I really don't see what its logic would be or what it clarifies. See also two other documents on MTA ("WDA"). See also the problematic chapter on "WDA" in "Handboek Wiskundedidactiek" (2012).

Definition and testing of insight

We better establish what we mean by insight.

The objective of "let students think" was already known with the thesis by Pierre van Hiele (1957), *"De problematiek van het inzicht".* [20] His thesis meant:

(i) The end to the traditional teaching of Euclid in Euclid's manner.
(ii) Henceforth a reliance on the Van Hiele levels of understanding (going from concrete to abstract, or, in levels: going from concreteness, to informal formulas, to formal formulas). [21]
(iii) The Van Hiele levels are a key discovery in epistemology. [22] Unfortunately, it is seldomly recognised that the Van Hiele levels have such key position. [23]
(iv) A bit implicit: Learning also goes from vague to precise.
(v) There are basically two types (extremes) of questions: Either exercises of what is already known and that allow one to practice or master what is known, or questions and problems that are chosen to support a *level shift* to a higher level of understanding (insight). (Exercising without insight would be drilling.)
(vi) Within the learning process, ordening is a basic step. This has evolved into the Joop van Dormolen OSAEV process. [24]
(vii) It is important to define the concepts (so that also students understand what is expected): abstraction means: leaving out aspects. (Not "something higher".) [25]

Van Hiele remarked that we test insight by presenting students with a new (type of) problem. See Table 1, right column.

- It would be helpful for students to know whether they have acquired that insight. Thus, provide them with such options for personal tests.
- But, insight is not something that you can develop a reasonable expectation about, and prepare for. Thus insight should not be part of the final examination in the national curriculum.
- Insight shows in the grade point average (GPA), and e.g. in (scientific) articles that researchers can write.
- One cannot say "the student did the test well because he or she showed insight" because it might as well be that the student was only lucky or happened to read a (better) book on the subject. (Testing is not an invitation to cheat.)
- There is a difference between insight and the **aspects** claimed for MTA. While insight would not be in the final exam, perhaps elements of aspects of MTA can still be included. This is a **valid approach** indeed. Like: Does the student give a structured list of properties ? Does the student present a model (define it as such, develop it properly, apply it as a model) ? These might seem to be a valid addition to the test on *knowledge* and *skill* (while we do not test *attitude* either). Unfortunately, cTWO does not provide teachers and test designers with tools to translate the laudable aims into practice.

[20] https://en.wikipedia.org/wiki/Van_Hiele_model
[21] https://boycottholland.wordpress.com/2015/09/08/pierre-van-hiele-and-epistemology
[22] https://boycottholland.wordpress.com/2015/09/08/pierre-van-hiele-and-epistemology
[23] See below for the misrepresentation in "realistic mathematics education" (RME), and see how Ben Wilbrink catalogues Van Hiele as part of RME: http://thomascool.eu/Papers/Math/2015-09-15-Breach-by-Jan-van-de-Craats-and-Ben-Wilbrink-wrt-scientific-integrity.html
[24] http://www.fisme.science.uu.nl/wiki/index.php/OSAEV
[25] https://boycottholland.wordpress.com/2015/05/23/abstraction-vs-eugene-wigner-edward-frenkel

Crucial misunderstandings and math war in Holland

Perhaps we should be glad that there is a continued effort at implementation of age-old objectives of teaching for insight. Education is working with (young) people and we can assume that this will never be perfect or finished.

However, we can still observe some misunderstanding about what the Van Hiele theory actually means.

- Apparently we are still suffering from Freudenthal's malconduct. [26]
- At the website of the *Freudenthal Head in the Clouds Realistic Mathematics Institute* (FHCRMI) there is a worrying discussion with various confusions of the *relationship of MTA and "21st century skills",* with a more or less implicit link to "realistic mathematics education" (RME) itself. [27]

It is also useful to be aware of the "math war" in Holland between "realistic mathematics education" (RME) and "traditional mathematics education" (TME). This is not about the aims of teaching but about didactics or the method of teaching. These are *ideologies* since there is no scientific corroboration or interest in this. There is also the *scientific* approach, that I have been calling "neoclassical mathematics education" (NME), that finds that RME and TME do not present mathematics but only so-called "mathematics", that contains the wreckage of 5000 years of wrong didactics.

Structure of this memo

Let me discuss some problems with and questions about this MTA.

Before doing so, it appears enlightening: (1) to consider two examples of what would be MTA, (2) to recall why cTWO was created in 2004 (ten·two years ago) in the first place.

The two examples of MTA seem to be taken from the earlier programme. MTA might emphasize what cTWO considered useful in the past. (This fits the statistics in the Buitenhuis (2016) master thesis.)

We must first establish some *common notions* before we can discuss the examples.

Disclaimers are:

- Some of my didactic principles are: (1) Integration of language, graphics, numerical tables, formulas, (2) Van Hiele levels of understanding (from concrete to abstract, and from vague to precise), (3) Wise use of computer algebra, [28] [29] (4) Agreement with students: mathematics is interesting and useful, but for science (e.g. econometrics) it is only one of the aspects and not something necessarily useful to specialise in (it depends upon one's preferences).
- I have spent only a limited amount of time on this notion of MTA. One reason is that the notion of MTA appears to be rather confused and problematic.

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[26] https://arxiv.org/abs/1408.1930
[27] http://www.fisme.science.uu.nl/wiki/index.php/21ste_eeuwse_vaardigheden_en_WDA's
[28] "Elegance with Substance" (EWS) (2009, 2015) and "Conquest of the Plane" (COTP) (2011) and "A child wants nice and no mean numbers" (CWNN) (2015), see http://thomascool.eu/Papers/Math/Index.html
[29] https://boycottholland.wordpress.com/2015/12/08/computer-algebra-is-a-revolution-but-21st-century-skills-q

Common notions: Van Hiele levels, science, applied and pure mathematics

Relevant concepts are: [30]

(1) There are science, "applied mathematics" and "(pure) mathematics".
(2) The defining property for ***mathematics*** is *deduction*, with definitions, theorems and proofs.
(3) The working horse in theorem proving in highschool is: write down all that is given for the problem, *including* (this is the trick) *the assumption that the theorem is true*, then structure the information, and the pieces should drop together. (Research mathematics is more complex.)
(4) The defining property for ***mathematics education*** are the Van Hiele levels, [31] that work from concrete to abstract, and from vague to precise, in which the first level is not chosen at random but targeted at the highest level of proofs.
NB. In "realistic mathematics education" (RME) it is claimed that it respects the Van Hiele levels, but its notion of "realistic contexts" actually contain a confusion between concrete (before learning) and applied mathematics (after learning). Hans Freudenthal (1905-1990) was an abstract thinking mathematician and no trained or practicing teacher of mathematics.
(5) ***Knowledge transfer*** (say from mathematics to physics) can only occur when there is knowledge in the first place. In mathematics, the knowledge consists of abstract theorems and proofs. The only way to gain that knowledge is to rise to the proper Van Hiele level. After that, the transfer becomes "applied mathematics". [32] See Table 1 for a key confusion. Beware however of mathematicians doing "applied mathematics", because they might not know about the relevant science, and create various disasters. [33]
(6) The notion of a ***model*** indeed seems an advance of the 1850s over the past. Euclidean and non-Euclidean geometry are models. The development of logic in the 1930s created more insight in model properties: systems of axioms have metamathematical properties, like completeness or whether they fit the intended application (say arithmetic). [34] For highschool education it is advisable to introduce students to thinking in terms of models. "What is the model that you use, why would you think that it applies to your problem, and what are actually the properties of that model ?" Scientific models differ from mathematical models. Scientific models might have rough definitions and be numerical, while mathematicians claim that their models would be pure and perfect. In school education we are still busy with the level transitions from concrete to abstract with Euclid and Descartes as the highest level.
(a) My impression is that the idea of modeling now is dropped into this basket without much thought (but I did not delve into what is said about this aspect).
(b) However, when you are modeling, then you are modeling something. For RME, they call this "context". [35] However, we should not confuse the *misuse of contexts*, about which there is criticism, and the *valid use of cases* that one wishes to model.

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[30] https://boycottholland.wordpress.com/2016/01/24/graphical-displays-about-the-math-war
[31] https://en.wikipedia.org/wiki/Van_Hiele_model
[32] It would be different in trade school, when the students might not be interested in the highest Van Hiele level, but do want to learn how to calculate VAT and so on. This is a different teaching objective than at VWO and HAVO.
[33] http://www.thomascool.eu/Papers/VTFD/Index.html and Dutch http://thomascool.eu/Thomas/Nederlands/Wetenschap/Artikelen/2013-02-14-PasOpMetWiskundeOverVerkiezingen.html
[34] Potentially, this might be seen as one level higher than what Van Hiele called the highest level of abstraction: Euclidean geometry. However, at that level, we might return to base, argue that Euclidean and Non-Euclidean geometry are at the concrete level, and then proceed in steps to more abstract metamathematical properties and theorems.
[35] "Essentieel in het modelleren is de rol van de context, waar het wiskundig model een min of meer adequate beschrijving van moet geven." p3, http://www.slo.nl/downloads/2014/onderwijzen-en-toetsen-van-wiskundige-denkactiviteiten.pdf

Table 1. First mathematics, then applications (FMTA): Proper versus so-called "FMTA"

So-called "FMTA"	*Proper FMTA*
	Use a problem to tease interest. This can be a pure or applied issue, depending upon the aim.
Introducting mathematics concepts and techniques	Introduce concepts and techniques, also with reference to the problem.
Exercises	Develop the mathematical core.
Application	Application: show how the original problem is tackled using the new tools.
	Eexercises on the mathematical core.
	Test the command of mathematics with a new application.
	Return to the value of mathematics as a way to establish certainty in knowledge. ("Mathesis" = "What is learned".) Distinguish the context of discovery from the context of formalising as definition, theorem, proof.

Example 1: Line of sight

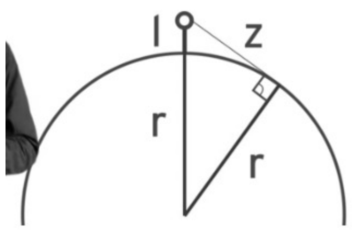
In this video, [36] [37] Jos Tolboom (SLO) presents the notion of MTA and gives "a problem", also calling it "an example". He does not explicitly state that this is an example of an MTA, but I presume this from the context.

The question (minute 1:40): "How far can you look on Earth ?"

He confirms that the answer starts with structuring, and that abstraction sets in early. One takes a circle (no mountains and such). Subsequently, there quickly arises a model. Key mathematical properties are that the line of sight *z* is a tangent, that such a tangent is perpendicular to the radius *r*, and that there is the Pythagorean theorem. The rest is algebra, with formulas and solution method. He regards it as "logic and proof" to see the effect of different lengths *l*.

Figure 1. Line of sight, drawing and formula

$$r^2 + z^2 = (r+l)^2$$



Tolboom suggests (minute 3:15) that one "looks up" the radius of Earth *r*, plugs in the length of the person *l*, and then can find *z*, which is the answer to the question.

His *intention* with this video (closing statement) is to show that *all* ***aspects*** *of MTA are included* in this "small example".

[36] https://youtu.be/0l9hrFHNw3Q?t=193
[37] http://www.platformwiskunde.nl/onderwijs_examenprogrammas_vanaf_2015.htm

Perhaps this purpose is well served, though one might discuss the aspect of logic and proof.

However, if this is an example of MTA, then: *is it a good way to teach mathematics* ? [38]

Some critical comments are:

(1) "How far can you look on Earth ?" is an (applied) science (or engineering) question, taken from geodesy. [39] The answer depends upon the local situation, e.g. whether you are on a mountain or in valley, or in fog, and who you actually are (and can use a telescope). A student might apologise for not being able to calculate the proper distance along the curvature but only find an estimate on a *line of sight* using Newtonian physics in which light is not affected by gravity. For (applied) science, reality matters, and for mathematics one abstracts from reality. Potentially, the student takes a Flat Earth as a fair approximation, and then z depends upon other parameters: and why would this not be a good model for the purposes of *mathematical thinking* ? (In this case, one might also look upward, into the universe, but we are assuming that it would be clear from context that one intends to look at the horizon.)
(2) "How far can you look on Earth *on average* ?" is an (applied) science question too, and calls attention to what you are averaging for: locations, circumstances, people ? We should not confuse *generalisation* with *abstraction.* [40]
(3) For highschool, we might take (1) and (2) as the basic Van Hiele level. Since the purpose is a mathematics and no physics class, the case would be taken as a *"framing",* say for at most 2 minutes at the beginning of class (more than in the video).
(4) For highschool, we would take the definitions, theorems and proofs of tangency, perpendicularity and the Pythagorean Theorem as the highest Van Hiele level.
(5) For highschool, we would take an intermediate Van Hiele level as the case in which students can "understand" the model as *given* in **Figure 1**, use the theorems, and do the algebra, to answer questions like "solve for z given r and l". (A student can use the Pythagorean Theorem without being able to prove it, e.g. having forgotten the proof from when it was presented.)
(6) It is a choice *within "applied mathematics"* to reduce the complexity in (1) and (2) to such a *mathematical model,* that we can apply (4), for a model as in (5). This is a very particular choice. There is nothing in the question "How far can you look on Earth ?" itself that suggests that *this particular choice for the model* must be made. It is a scientific fact that Earth has seas, mountains, vallies, clouds and fog. It is not obvious how to abstract from these. (How did they determine Earth's radius in wikipedia ? An estimate of masses and densities might make an estimate of volume, and result in an estimated sphere. Or, did they use some average circumference divided by Θ = 2π ?) But lines of sight might still be hampered by the average cloud. Also recall the option of a Flat Earth. Thus, it is a discussion *within "applied mathematics"* that this particular model serves the purposes of the exercise.
(7) Thus, there is no information in the question itself *that one can use an estimate* of r. A student confronted with the question "How far can you look on Earth ?" without this additional piece of information that r can be taken *as given* would be lost. (Getting information about a value of r or merely "assume that r is known" would be an indication as well about how to proceed with modeling.)
(8) A better question for testing on mathematics is:
"Assume that the Earth is a perfect sphere. You are standing on that sphere. You have perfect vision and nothing obstructs your view. You have measured your *line of sight* to the horizon. What is the radius of that sphere, given these data ?" [41]
This question has the same *intended* mathematical content, and removes the noise from

[38] It is a question whether this example has been used in the pilots and how it worked for students.
[39] https://en.wikipedia.org/wiki/Geodesy
[40] https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson
[41] (i) Notice the shift of focus. (ii) The concept of "line of sight" might need an explanation. One might say "like the bird flies" but birds tend to follow the curvature of the Earth too. One might refer to light but see Newton vs Einstein, and thus one needs to explain that this involves a line segment. (iii) Do not ask "Give the radius of that sphere", for any number then would suffice, and the answer really concerns the relation. A question like "Give the best estimate of the radius" would involve estimation while this concerns a pure algebraic outcome.

(1), (2), (6), (7) and the issue of looking upwards into the universe. One doesn't assume that the radius of Earth is given, but one uses the notion that one can measure how far one can look.

(9) A subsequent question is: "Can this model in (8) be used for estimating the radius of the real Earth ?" Its answer would be that it might be an instance of applied mathematics but insufficient for (applied) science.

(10) Observe that MTA apparently wants that students are able to introduce key variables *r, l* and *z.* This assumption is maintained in the "better question" in (8). It is still an issue whether this is wise. There are two learning goals:
(a) introducing variables,
(b) making a useful model with those variables.
It is definitely *science* to do both (a) and (b). Only (b) would be proper (*deductive*) mathematics (with given conditions so that it is not science).
If one wants to test (b) only, for this particular model, then the question would be: "Assume that the Earth is a perfect sphere with radius *r*. You are standing on that sphere, with your eyes at height *l*. You have perfect vision and nothing obstructs your view. Your *line of sight* to the horizon has length *z*. What is the radius of that sphere, given these data ?"

(11) (i) The latter would be a mathematical question, using a proper Van Hiele context. The idea is that the context would help the student to make the transition from the basic Van Hiele level to the intermediate Van Hiele level. Teaching would be about: what do those variables mean, how would you draw them, how do they hang together, what properties do you know about them and their relations ? (ii) The student can only answer the question by making a mental image and possibily support it by a drawing like **Figure 1**. For this, the student must create the line segments for *r, l* and *z.* Thus there is a serious aspect of *creation* involved.

(12) However, when (a) is included in the question – having (8) instead of (10) – then the mathematical examiner requires that the student can also *create* the very variables. A competent quantitative student will remember the properties of a *sphere,* and introduce the variable *r,* and work from there. However, one should not test (b) by also including (a) as a possible bottleneck.
Of course, in mathematical modeling one can introduce variables. But I reject the idea that the question "How far can you look on Earth ?" *as such* can be used to test mathematical competence on both (a) and (b).
Overall, my suggestion is that there should at least be a separate chapter with explanation, training and testing on (a), i.e. learning to interprete texts quantitatively and introducing variables, and a chapter on (b), using a model, *before* combining (a) and (b).

(13) Overall, this example context presumes the mathematical theorems in (4), and thus can be seen as "applied mathematics", in which mathematical results are applied to the real world. The very question "How far can you look on Earth ?" is a teaser only, see (3). The test question format, with (8) more complex than (10), comes at late stage of use. The example thus can be used at different stages in education for different *though limited* purposes.

In Van Hiele's approach, (10) is a concrete example [42] (and (8) more opaque), that allows one to stimulate a level transition towards abstract theorems and proofs in (4). Obviously, this is what mathematics is about: working towards theorems and proofs.

The question "How far can you look on Earth ?" suffers from the same confusion as "realistic mathematics education" (RME). Van Hiele's argument (that learning is from concrete to abstract) is misinterpreted as the argument that learning should start with "realistic contexts". [43]

(a) Concreteness (before learning) is confused with applied mathematics (after learning).
(b) For Van Hiele the cases should be concrete, and targeted at a level transition, while "realistic contexts" are given often *with distractions*, also with the arguments about

[42] It is concrete, in that it helps students understand how the situation arises. Why would one extend the radius with some length *l,* and then draw a tangent from there ? Without this context, the problem as such seems without content, and thus, for many, without interest.
[43] "Conquest of the Plane" (2011) Chapter 15.

making students feel at home and about "knowledge transfer" (what is learned in mathematics can also be applied elsewere, e.g. physics): and thus need not be targeted at level transition.

The question "How far can you look on Earth ?" is a question from geodesy, and at best one of applied mathematics (after learning). It can be used as a teaser (before learning) as in (3), but not as a concrete stepping stone for a Van Hiele level transition. For such, the question must be edited as we have seen.

This example of MTA thus apparently contains all aspects of MTA, but is didactically confused, at least, if it were intended for application in class. No doubt, students in class might enjoy watching the video and see how mathematics can be applied, but then we are speaking about (3), and it gives away all the answers, likely without having them think deeply.

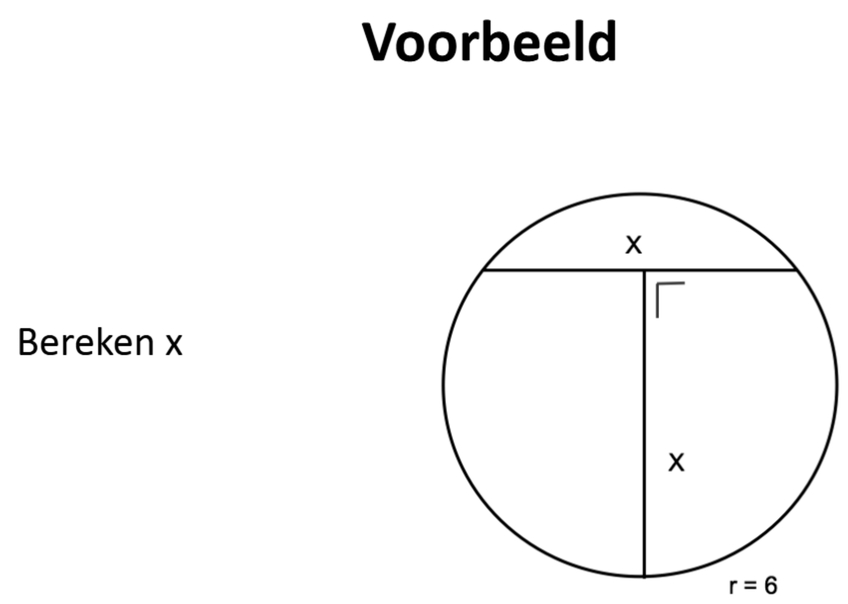
If the question is intended as a test question, then the aspects that Tolboom recognises in the video must be linked to scores in the grading. Till now this is lacking.

If the video is intended as a suggestion to teachers how to handle the MTA, then it lacks the discussion above, and would suffice only as an overview.

Example 2: Equal perpendicular line segments in a circle

Peter van Wijk (SLO), presents various MTA, of which this sheet 4, with Figure 2 and question: "Calculate *x*." [44] [45]

Figure 2. Calculate *x*



Some comments are:

(1) It is not stated or indicated that the vertical line segment *x* is at half of the horizontal *x*. For its proper purpose, one would require double dashes on both sides. Potentially, the designer might have thought that this might give away too much information. Students actually solving this will simply assume this, seduced by the symmetry in the drawing. Still, if they would not make a distinction between making an assumption and using a certainty (a *given*), then they would be in error.

[44] http://www.slo.nl/agenda/hier-havo
[45] http://downloads.slo.nl/Documenten/Presentatie%20Peter%20van%20Wijk.pdf

(2) The mentioning of *r* serves to indicate that the figure is a circle. (There is also some help provided by the very mentioning of *r* rather than saying that it is a circle.)
(3) This question can be presented to students who already have the Pythagorean theorem and who have learned about substitution, or conversely. What is tested here really ?
(4) This question has no context, though the $r = 6$ and "calculate" eliminate its pure mathematical potential. (At this level, this calculation should no longer be a mathematical challenge. It is no interesting result. Perhaps it is interesting to see whether students can solve the problem and still have energy to do a proper calculation ?) (Merely mentioning the right answer should not earn points since one must show the steps.)
(5) The mathematical core is the *existence proof* that such a situation is possible: Find the relation between *x* and the size of the circle (the radius). A question is: "Prove that *x* depends upon the size of the circle, and determine the actual relationship with a formula." This does't mention *r.* The proof would consist of giving the formula and showing that it holds for any circle, i.e. that *r* is an independent variable. A more abstract question is: "There is a theorem on this. Find the theorem and prove it."
(6) Thus, the question would be applied mathematics if it would test the command of the Pythagorean Theorem and substitution, or it would be at the intermediate Van Hiele level if it is intended for a level transition towards formal reasoning. Perhaps MTA have this ambiguity by themselves ? However, it is not clear to me what the next step towards formal reasoning would be, and thus is seems more like applied mathematics.
(7) The student must create a line segment *r* from the center to one of the horizontal endpoints. Subsequently there must be a new variable $y = x – r$ to enter into the Pythagorean Theorem. (i) The introduction of a variable is a key step, now not in text but in geometry. This is very similar to the proof of Thales' Theorem, in which one must also draw a radius and then label the angles (and then use isoscelesness rather that the Pythagorean Theorem). The introduction of variables in problems of geometry seems easier and more natural than the introduction of variables for text questions (as in Example 1), that only later are translated into geometry. That this seems simpler might be from training of the teacher. We should be able to test whether it makes a difference when the vertical *x* is replaced with part *r* and part $x – r$. (ii) Potentially there are other solution approaches but I have not tried.
(8) Calculation issues, like checking an outcome using other data (like the given value of *r*), might be tested here too, but potentially other questions are more suited for such.

This is rather plain old (analytic) geometry. Not in the explicit form of *definitions, theorem, proof,* but still with that (hidden) structure. This question was also possible in the old programme, even though analytic geometry was no explicit objective. In the past, we might have had:
"Given is a circle that contains two equal line segments of length *x,* that are perpendicular, with one at half of the other of these line segments. A theorem is that such a situation is possible, and that *x* depends upon the size of the circle. Prove that theorem and determine the actual relationship with a formula. See the drawing. Use *r* for the radius."
More words indeed. This perhaps indicates that MTA might be a matter of putting more emphasis on this type of question, but in a loose manner, without explicit *definition, theorem, proof*. Perhaps the mentioning of a "theorem" kills curiousity, while "calculate" kindles the interest how this can be done ? For aspiring mathematicians it would work conversely.

It would be a suggestion to present students with the two variants, and let them discuss how they differ, so that they grow aware that a problem can be presented in various ways, giving "different models" to approach a situation. In such a situation, a teacher could also explain why mathematics has tended historically to prefer the *definition, theorem, proof* format.

Origin and timeline of cTWO: failing skills in arithmetic and algebra

Around 2004 there was concern about dwindling competence in mathematics. Had the highschool diploma been degraded ? Universities and colleges complained about the competence in algebra and even in arithmetic, and created their own entrance examinations. [46] In Holland, the idea is that secondary (SE) exit exam automatically should provide a tertiary (TE) entry. Thus a commission cTWO was created.

Somehow, they did not focus on the problem that was observed, but they decided to create a new curriculum. Or, perhaps, a new curriculum was already planned, and this was included.

- One potential cause was the reliance on the graphic calculator (GC). Secondary education (SE) might not maintain what students had already learned in elementary school (PE). [47]
- Another potential cause was that students didn't actually learn arithmetic in primary education anyway, [48] so that secondary schools were forced to use the CG. Thus a SLO commission (Meijerink) was created with a report in 2008, and a KNAW commission (Lenstra) was created with a report in 2009. [49]
- Around 2004, "realistic mathematics education" (RME) started getting criticism from "traditional mathematics education" (TME). [50] A key criticism was that RME uses **context questions** that rely on verbal understanding and that do not focus on mathematics itself. Below, we will discuss that RME and TME appear to be **ideologies and not science**. There actually is a "math war". [51] [52] This war continues to this day, as is also expressed by Henk Broer (2015:28), chairmain of the mathematics section at KNAW. [53] [54] Since a few years I have been labeling the **scientific approach** as "neoclassical mathematics education" (NME). [55]
- Problems with new "profiles" and the protest that cTWO did not really solve the gap between SE and TE caused the minister of education to create a "resonsansgroep" in 2006-2007, headed by Jan van de Craats. [56] [57]

For the timeline:

- A commission headed by Jeroen Dijsselbloem concluded in 2008 that the government determines *What* must be learned and that teachers decide *How* this is done. [58]
- Following the report Meijerink of 2008, the minister of education decided to create a separate **arithmetic test ("rekentoets")** for all "streams", including secondary

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[46] Universities and colleges may not be so quick to act. The problem had been festering for years. Universities and colleges already had entrance examinations for students with deficient preparations. Thus they simply decided that these examinations would apply to all students. However, taking this "simple decision" was actually quite a step, both psychologically and bureaucratically.
[47] This document http://downloads.slo.nl/Repository/afstemming-wiskunde-natuurkunde-tweede-fase.pdf p67-68 gives a warm support for the (wise) use of the GC. Its Appendix 11 has been copied in this document below as **Appendix E.** In my analysis, education should have adopted computer algebra already around 1995. The present delay of 16-20 years however is a blink in the history of human civilisation.
[48] http://www.nieuwarchief.nl/serie5/pdf/naw5-2007-08-2-132.pdf
[49] http://www.nieuwarchief.nl/serie5/pdf/naw5-2010-11-3-173.pdf
[50] https://boycottholland.wordpress.com/2016/01/24/graphical-displays-about-the-math-war
[51] https://en.wikipedia.org/wiki/Math_wars
[52] http://vorige.nrc.nl//opinie/article2039486.ece/Realistisch_rekenen_niet_goed_Kinderen_presteren_juist_beter Saillant detail: Diederik Stapel is one of the signatories.
[53] http://www.math.rug.nl/~broer/uploads/Main/grede.pdf
[54] http://thomascool.eu/Papers/AardigeGetallen/2016-10-05-Afscheidsrede-Henk-Broer-en-diens-KNAW-sectie-voorzitterschap.pdf
[55] https://boycottholland.wordpress.com/2016/01/24/graphical-displays-about-the-math-war
[56] https://staff.science.uva.nl/j.vandecraats/resonansgroep.htm
[57] http://thomascool.eu/Papers/Math/2015-09-15-Breach-by-Jan-van-de-Craats-and-Ben-Wilbrink-wrt-scientific-integrity.html
[58] https://nl.wikipedia.org/wiki/Parlementair_onderzoek_onderwijsvernieuwingen

education.[59] The latter was a political decision and against the preference of SE teachers. The test appears to be a continuous drama. Pre-university students are doing well but might not need the test anyway. Pre-college students are not doing well but might only lack "motivation". The drama is with the trade schools (MBO). The problem actually lies in primary education, see the following.

- The KNAW report of 2009 on arithmetic in primary education claimed that they could see no statistical difference in the results between "realistic mathematics education" (RME) and "traditional mathematics education" (TME). *This interpretation must have been important for cTWO's view on RME versus TME.* However, psychometricians working for the KNAW report of 2009 conducted an invalid test, did not spot the true problem, and breached integrity of science by refusing to answer to relevant questions. [60] For arithmetic in primary education (PE), traditional algorithms are important for algebra later in secondary education (SE). [61] We can only suppose that cTWO would have looked differently at the issue of RME versus TME, if and when KNAW in 2009 would have reported that it could see a difference on this issue. This particular finding applies for the transfer from PE to SE, but this would have given an alert for the transfer from SE to TE.
- The idea of cTWO was to start in 2010 but there was delay. [62] [63] One aspect of the delay is that the issue of arithmetic apparently wasn't solved in 2010 and still features as a *separate test* on arithmetic in the new exams (though not part of the cTWO package).
- I myself started publishing on mathematics education from 2008 onwards, starting with an advice for a parliamentarian enquiry and the creation of a "Simon Stevin Institute". Thus **there were at least 4 years in 2008-2012 during which my results might have had some influence on cTWO and the current mathematics education programme.** [64] This is relevant to observe, for I wondered whether I might only have had the simple bad luck of presenting innovation while everyone else was focussed on the cTWO programme. However, there has been ample time to respond. Also, this influence was directly blocked. [65] Also, Parliament asks for mathematics (the *What*) but my diagnosis is that it gets so-called "mathematics". My analysis in 2009 also pointed to another deficiency in the KNAW 2009 report, namely the treatment of fractions and the pronunciation of numbers. Since 2015 there is also the diagnosis about the error by the psychometricians.

Problem 1. No news on aims, so the crux lies in implementation ?

Problem 1 with MTA is that its aim of insight is an *age-old objective*. Indeed, we can read that cTWO claims that there is nothing new under the sun. Thus, why the *hyberbole* on *new* ?

Subsequently, *some* drilling is useful. Once you understand the tables of multiplication, it still is useful to put them to memory, for you really need to have this automated when you want to work at ease on more complex problems later on. Also, one must learn the algorithms of arithmetic in PE so that one can do algebra in SE. It is less clear though what these dependencies are between SE and TE. Once you understand the quadratic function then it is useful to do some drilling, to reduce memory overload for more complex issues. [66]

Perhaps the advance by cTWO lies in the *reformulation* and *more explicit targeting* ? However, why would this be so ? Is cTWO's argument that earlier authors were not clear enough ? If it already was a target for earlier programmes, then the real advance should rather come from *implementation*.

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[59] http://www.taalenrekenen.nl/referentiekader/betekenis/FAQ/voortgezet
[60] http://thomascool.eu/Papers/Math/CWI-Leiden/2016-09-30-Letter-to-CWI-anonimised.pdf
[61] https://boycottholland.wordpress.com/2015/12/18/algebra-is-a-troubling-word
[62] http://www.nieuwarchief.nl/serie5/pdf/naw5-2009-10-1-029.pdf
[63] http://www.trouw.nl/tr/nl/5009/Archief/article/detail/1874375/2011/04/13/Hoezo-eigenlijk-wiskunde.dhtml This article mistakes mathematicians as also competent in mathematics education and teaching. These however are quite different specialisations.
[64] http://www.thomascool.eu/Papers/Math/2009-10-15-Reacties.pdf
[65] http://thomascool.eu/Papers/Math/2013-02-06-Colignatus-nav-cTWO-Eindrapport.html or pdf at http://thomascool.eu/Papers/Math/2013-02-06-Colignatus-nav-cTWO-Eindrapport.pdf
[66] https://boycottholland.wordpress.com/2016/04/24/teaching-quadratic-functions-re-engineered

But the implementation is not so strong.

There are still various documents in 2016 *explaining to teachers* - as if they would not know how to teach mathematics – "how MTA are done and how MTA help in thinking mathematically". Really ? Did these authors not read Van Hiele or Van Dormolen ?

The aspects claimed for MTA – ordering, modeling, logic – might partly be didactics and partly be valid extensions of the learning goals of knowledge, skill and attitude. But we don't see them developed. There are examples of MTA but not how they are graded (on these aspects of MTA). Buitenhuis (2016) is a first effort at creating clarity, but not necessarily convincing.

The true situation might be different. Very likely, the new programme came about *partly* as a *compromise* between different groups, like RME and TME. They could agree on aims but less on implementation. The article in Trouw referred to above is indicative. [67] Perhaps some interviews with key agents could confirm this. (Preferably by a parliamentarian enquiry.)

Also, the real problem in mathematics education is rather the difference between mathematics and "mathematics", see neoclassical mathematics education (NME). But when you are warring with one-another then it is difficult to see other things.

Problem 2. The order of aspects or competences

cTWO presents the aspects or compentences in the order of (algebraic) modelling, ordening / structuring, analytical thinking and problem solving, manipulating formulas, abstraction, and logical reasoning and proving.

However, an order that makes more sense for the empirical sciences [68] and the Van Hiele levels of understanding are: (i) ordening / structuring, abstraction, (ii) (algebraic) modelling, manipulating formulas, and (iii) logical reasoning and proving. Let me refer also, as said, to Van Dormolen, OSAEV. [69]

The whole might be called analytical thinking and problem solving. (This is not to be confused with the names of analytic vs synthetic geometry.)

Problem 3. The name is self-contradictory

I regard *thinking* as opposed to *doing*. Sit on your hands and think about an issue before you start acting.

Now, however, there is a "think-activity". (And this would be something for mathematics, that requires thought-eye-hand co-ordination when dealing with texts, graphics, tables and formulas.)

An **oxymoron** has self-contradiction [70] but wikipedia claims that there might be a concealed point of truth. [71] For us it suffices to say that the term is self-contradictory.

My impression is that the cTWO-people who thought up the phrase "MTA", or allowed this expression, have little sensitivity for language.

However, the issue is not really one of linguistic sensitivity.

When the aim is age-old, then there is no need for a new label. It would suffice to use the old labels, and every teacher would know what one is referring to.

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[67] http://www.trouw.nl/tr/nl/5009/Archief/article/detail/1874375/2011/04/13/Hoezo-eigenlijk-wiskunde.dhtml
[68] https://en.wikipedia.org/wiki/Empirical_research#Empirical_cycle
[69] http://www.fisme.science.uu.nl/wiki/index.php/OSAEV
[70] http://www.merriam-webster.com/dictionary/oxymoron
[71] https://en.wikipedia.org/wiki/Oxymoron

For example: less reliance on the GC and more time for thinking (with pen and paper).

Thus, the very fact of a new name (remarkable because of its self-contradiction) suggests that something else is happening.

Problem 4. The math war in Holland. MTA looks like the use of contexts in "realistic mathematics education" (RME)

MTA looks very much like the use of contexts in "realistic mathematics education" (RME).

Indicative is this text on page 34 of "*Rijk aan betekenis*" (2007). [72]

> "Deze kijk op de rol van contexten sluit aan bij de **theorie van realistisch wiskundeonderwijs,** die de afgelopen decennia in Nederland school heeft gemaakt, maar waarvan de verwezenlijking in de praktijk van het voortgezet onderwijs weerbarstig blijkt te zijn. Ook correspondeert deze visie op het gebruik van contexten met de zogeheten context-concept benadering, die bij de huidige vernieuwing van de **exacte vakken** als uitgangspunt dient. Daarnaast wijzen ook onderzoekers op het belang van wat 'situated cognition' wordt genoemd (Noss en Hoyles, 1996)." [My emphases]

Observe that MTA is actually alternative to the structured approach in Table 1, right column:

- applied mathematics
- proper mathematics, with deduction and proving theorems
- Van Hiele level transitions using adequate problems.

A possibility is that cTWO doesn't like the first two since they do not fit the RME ideology, and that both cTWO and RME are confused about the Van Hiele level transitions.

A possibility is that ideologues of RME in cTWO want to defend the use of "realistic contexts" against the criticism of TME ideologues also in cTWO. Perhaps the TME reasoned: let them have their context, as long as the exams get tougher ?

My fear is that MTA can be understood as a ploy from RME to simply *rename* the contexts, and sell the same ideology but now under this new name of MTA. When you introduce a new name as something that you want to do, *then you don't have to say what you don't want to do*. (When you don't like to go to Madrid, you can say positively: Let us go to Paris !) In that case the TME have been sleeping (or thinking abstractly and not realistically).

Also, a new term, presented from a position of power, forces everyone to use that term, also in criticism, so that many people (outside the core circle) might think that it really is about something. Researchers feel themselves forced to look into the use of the new term to see whether the term really means something. When a term becomes worn-out, then it is easier for people to disregard it, since they know that it is rather obsolete. But a new term for the same obsolete idea gives it a new life.

NB 1. Everyone can observe that there is a "math war" [73] between RME and TME. These are not scientific positions but derive from ideology. This war actually exists also in other countries, e.g. see the issue on "*reform math*" [74] [75] and e.g. the case of Jo Boaler. [76] [77] [78] [79]

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[72] http://www.fi.uu.nl/ctwo/publicaties/docs/Rijkaanbetekenisweb.pdf
[73] https://en.wikipedia.org/wiki/Math_wars
[74] http://opinionator.blogs.nytimes.com/2013/06/16/the-faulty-logic-of-the-math-wars/?_r=0
[75] http://www.nychold.com
[76] http://www.howtolearn.com/2015/02/drilling-and-rote-memorization-is-less-effective-for-learning-math
[77] https://educationrealist.wordpress.com/2013/01/16/jo-boalers-railside-study-the-schools-identified-kind-of
[78] https://gregashman.wordpress.com/2015/09/19/jo-boaler-is-wrong-about-multiplication-tables

- TME is an ideology of mostly mathematics professors who have no qualification for didactics in primary and secondary education.
- RME, with that particular name, is an ideology started from Hans Freudenthal (1905-1990). He was an abstract thinking mathematics professor too, who saw his intellectual capacities for mathematics dwindling, who gave himself the choice to continue in either education or history, and who chose education. Freudenthal had no training in education or didactics, but joined up with pedagogues and educators who welcomed a mathematics professor who *apparently* was willing not to pose strict demands upon understanding, skills and attitude on mathematics in traditional manner. (Freudenthal's wife Suus Lutter introduced the Jenaplan schools in Holland. [80] [81])
- Reform math. Obviously, it would give Freudenthal too much credit for causing the current world math war, and there will be various other influences in the various countries, giving the general label "reform math".
- However, it is useful to focus on RME since it has been developed as an ideology, with also a *Freudenthal Head in the Clouds Realistic Mathematics Institute* (FHCRMI), now at University of Utrecht, but as an ideological institute it should not be at a university.

NB 2. Above quote from page 34 refers to the use of contexts in the (exact) sciences (physics, biology, etcetera).

- This use of "contexts" by science education is logical, since they deal with science cases and not with mathematics. It is a logical fallacy that their use for science is presented as a corroboration for the didactics of mathematics. If the same linguistic expressions are used, then this is because of involvement of FHCRMI.
- FHCRMI has now merged with other STE into a University Utrecht institute for STEM. The sciences are easy victims for RME. They provide contexts, and they might like the attention of mathematics for their contexts. But *they are not qualified for mathematics education,* and are not able to diagnose that RME is an ideology and no science. Thus, they are unable to see how they are abused *by giving RME more credibility.* (Indeed, cTWO argues (with a fallacy) that their involvement is another proof for RME.)

NB 3. Above quote from page 34 refers to 'situated cognition' and Noss andd Hoyles (1996). [82] [83] The statement seems like a *magical formula*. In a scientific text, one would explain the issue, and provide the reference for further study. Now cTWO presents a big mystery only, with the *simsalabim* of a reference. Fortunately, there is wikipedia – no source but a portal – that has this entry: [84]

- "Situated cognition is a theory that posits that knowing is inseparable from doing, by arguing that all knowledge is situated in activity bound to social, cultural and physical contexts." (My example: Adults put in classroom seats start behaving as students.)
- Also: "(...) learning is seen in terms of an individual's increasingly effective performance across situations rather than in terms of an accumulation of knowledge (...)".
- However, that page does not refer to Noss and Hoyles (1996).

Thus, this appears to be an epistemological and / or psychological *theory*. It is not necessarily scientifically proven. It all depends how one defines thinking and acting. We should hope that cTWO does not take this theory for a fact, and that this is the basis of the new programme.

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[79] http://math.stanford.edu/~milgram/Jo-Boaler-reveals-attacks-AccusationsResponse-trans.html
[80] https://nl.wikipedia.org/wiki/Suus_Freudenthal-Lutter
[81] http://www.deoudrotterdammer.nl/archief/dou/2014/oudutrechterweek06_2014/files/assets/common/downloads/page0001.pdf
[82] http://link.springer.com/book/10.1007%2F978-94-009-1696-8
[83] http://www.lkl.ac.uk/cms/index.php?option=com_comprofiler&task=userProfile&user=108&Itemid=111
[84] https://en.wikipedia.org/wiki/Situated_cognition

NB 4. A new phrase on p35 is "authentic context". What is this ? **Ideologues** are apt at creating new phrases as if there would be a new idea, and if criticism on the ideology is outdated because there is this new term.

Problem 5. Testing doesn't appear to be testing

Overview

Not only the design but also the testing of MTA is problematic.

There is a document by Anne van Streun (2014), on teaching and testing MTA. [85]

- Above, MTA has appeared to be a fairly confused concept. When Van Streun starts explaining *how to teach and test it*, then confusion increases.
- It also appears that cTWO presented its final report in 2012, while implementation is still being worked on.

The scheme of *design, pilots, final report* suggests that cTWO presents a completed package. However, there are *loose ends all-over.* Input and output cause too many questions. The *aims are confused,* the *exam criteria have not adequately been tested*.

- When you teach without knowing what will be tested, then this is unprofessional.
- When you design a national exam standard and don't know what will be tested, then this boggles the mind.
- This is not *evidence based education.* Again something is being introduced from some ideology without proper empirical testing first.
- This is not mathematics but "mathematics", see *Elegance with Substance* (2009, 2015).

This might actually be the way how national exam standards have been created in Holland traditionally, so that the participants in this process cannot imagine how to do things differently.

Potentially, the nitty gritty appears in the subsequent documents when examiners sit down and decide what they will actually be testing. [86]

- However, why this layer of confusion, when supposedly the real documents should be found elsewhere ?
- If an examiner presents an exam question claiming that it tests MTA, *how are we to know for sure* ?
- Or will there be a declaration from a power position: "It is MTA because we exam question designers think it is." And then "take it or leave it" ?

More confusion

(1) On page 3, all of a sudden, without any substantiation, Van Streun declares, ex cathedra, that "problem solving" and "abstraction" would be the key notions.

> "Bekijken we op enige afstand het gehele complex van wiskundige denkactiviteiten, dan valt op dat er twee hoofdlijnen zijn: probleemoplossen en abstraheren."

One can imagine that the MTA package is large so that one might desire a simplification, but why precisely these two ? Well, if MTA actually means problem solving with use of some abstraction (rather than cutting the knot, as Alexander the Great did), then Van Streun has merely paraphrased MTA. How are we to know ? Why can Dutch researchers in didactics of

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[85] "Onderwijzen en toetsen van wiskundige denkactiviteiten", http://www.slo.nl/downloads/2014/onderwijzen-en-toetsen-van-wiskundige-denkactiviteiten.pdf/download
[86] http://www.slo.nl/organisatie/recentepublicaties/veranderdwiskundeonderwijs

mathematics not provide a clear definition of MTA *and* apply this consistently ? Is the notion *that you are defining highschool examination standards* not important enough to focus ?

(2) On page 8, it is explained that the earlier cTWO publications (earlier before this document of 2014) *are inadequate* for the design and testing with exam questions:

> "De globale beschrijving van de zes typen WDA uit de documenten van de commissie Toekomst Wiskundeonderwijs (cTWO) geeft te weinig aanknopingspunten voor het ontwerpen van bijpassend onderwijs, het maken van geschikte opgaven en het opstellen van een toets of examen."

Thus cTWO in 2012 had only a rough idea, and also the pilots were fairly rough, so we are basically taking a splash into the deep water, where sharks swim ?

(3) It is claimed:

> "Ondanks de grote mate van consensus over de beoogde meerwaarde van het wiskundeonderwijs is het in het verleden niet goed gelukt om dat in termen van eindexamenprogramma's tot uitdrukking te laten komen." (page 9).

Is that so ? It might well be that the exam programmes in the past were okay (check my gymnasium beta exam of 1972), but that the inflow from primary education got lower quality (though HAVO-VWO still take the most academically minded students), and that the main problem has been loss of quality in implementation (like the use of the GC). Where is the *proof* ? (A simple reference should suffice !)

(4) A problematic statement is:

> "Het nieuwe woord wiskundige denkactiviteiten (WDA) dekt de verzameling van al die (lange termijn) onderwijsdoelen die de leerstofdoelen te boven gaan." (page 9).

Thus, (i) there are the aims that translate in lesson materials, (ii) MTA would *surpass* this (and not be in the lesson materials) ? Thus, students are tested on what they are not trained for ? It is okay when they don't know the questions themselves yet, but should they not have a reasonable expectation about what to expect, so that they can *prepare* ?

(5) Page 11 explains that the implementation *must still be developed*, *even though it already has been decided* that the examination will be on MTA (whatever that might be). There is no *implementation path* but only a *recommendation* of some general ideas:

> "De nieuwe examenprogramma's maken het noodzakelijk om het lesgeven en toetsen in het Nederlandse wiskundeonderwijs opnieuw tegen het licht te houden. In hoofdstuk 4 worden de ervaringen puntsgewijs samengevat in de vorm van aanbevelingen."

Thus, it is up to teachers now to start *asking questions* to their students, so that they *start to think mathematically,* so that they can pass in the highschool exams in 2017 (HAVO) and 2018 (VWO).

This sounds like a rather irresponsible approach.

Potentially, the pilots have indicated how far examinators can go with designing questions that are labeled as MTA. This turns everything into an exercise in bureaucracy: *MTA is what the exam developers create as MTA.*

(6) A bit cynical: when students appear not to know something, and have to think longer, then it is taken as MTA. On page 27:

> "Veel leerlingen bleken niet de parate kennis te hebben van puntsymmetrie anders dan in de oorsprong en moesten daarom eerst uitvogelen wat dat concreet voor het te bewijzen betekende. Dus WDA."

(7) And so on. With such easy findings of confusion, it leads too far to see whether the argument is sound or whether there might be a deeper intellectual problem hidden perhaps.

Chapter on testing doesn't discuss testing

The reader expects to see a discussion on testing, but there is none of that. There is a rather superficial discussion of "example MTA questions", and key issues are not addressed.

(1) If a student *has been thinking,* which is the objective of MTA, but *hasn't found the result*, then the test still gives .... points ? (Fill in the blanks.)

Potentially, the questions would be so diverse, that each test question comes with *its own set* of possible point-earners, for which there would be no general rule. But should students not be able to *prepare* ?

(2) *How are you going to test structuring ?* Suppose that students want some information about what they will be tested about. You tell them that structuring is one of the skills. Thus, a student presents you with a list of items concerning the question, that indeed shows some structure. The structure is irrelevant for the solution, but the student doesn't know the solution, and is still "thinking on it". No points ?

(3) *How are you going to test modeling ?* If the student presents a model, but the model doesn't fit the intended interpretation, then perhaps the competence in modeling is okay, and the student earns some points ? For example, the intended interpretation is exponential growth, but the student provides a nice linear model, and has really thought about it. Now, is this a question about modeling, or is it actually a question about recognising exponential growth ?

(4) Mutatis mutandis for the other claimed aspects of MTA.

Fairness in opportunities

Notice the potential imbalance in grading the exam.

Suppose that MTA covers 15% of the exam. Suppose that there would be 30% simple questions that all should be able to do and 55% tougher ones as well, but all more on "routine" instead of the MTA that wants you "to think".

A notion is: potentially a student doesn't need MTA to graduate. Focusing teaching on the "routine" questions, even the more difficult ones, might be the best strategy for the less competent students. However, their scope of succeeding now has been reduced by 15%.

- Mathematically competent students should be able to find solutions, and for them it makes a difference between say 7.5 and 10.
- Mathematically less competent students are unlikely to find solutions, and they have to earn their 5.5+ by using only 85% of the questions.
- In the past, these latter students (apparently) could hope for 100% with "routine". Answering the simple steps in the 70% more difficult questions might already earn 7% points too.

I don't know how the examiners saw this in the past and see this for the future, but I expected a discussion of this in this particular document, and don't find it.

Why test MTA in the exam itself ?

cTWO must have reasoned or taken as an axiom:

> *When we introduce MTA then it should also be tested in the final exam.*

(I haven't checked the earlier documents to see whether there is a considered statement on this. See also the confusing comment (2) above that those earlier documents would be inadequate.)

However, why should this be so ?

When cTWO has discovered a method to get students to think mathematically (it only needs to find an implementation though), then this competence will show itself in *all* questions.

The test and proof on thinking well consists of the rise of the Grade Point Average (GPA). (Though there is A.D. de Groot on "vijven en zessen".)

There really is no need to test MTA in the exam itself.

Teachers can explain to students that time spent on challenging exercises is time well spent, both for fun, and the overall learning, and the effect also on the more routine questions. It increases motivation on developing the routine when you see that you need it for challenging questions.

The real problem might be that mathematics teachers in Holland rely too much on the books, and that those books play it safe, focus on routine, and do not provide the intellectual challenge needed. This is an entirely different issue than the design of a new exam standard. Experts on didactics might have had a discussion with the authors of the textbooks to resolve the problem.

cTWO seems to have a particular philosophy of education, and wants to impose this via the exam standards. But exams should test competences of students and not philosophies of examinators or teachers (though test theorists know that it is difficult to separate these).

As said in the introduction (a statement at this spot was actually moved to the introduction): insight cannot be tested in the final exam, and likely only elements of aspects can. The latter would have been valid inclusions of the knowledge, skill and attitude. There should have been no confusion about the meaning of MTA, and there should have been swift implementation of test questions, before the final report was published.

The experts on didactics in cTWO should have known this, and thus there is an increased likelihood that they were no real experts but ideologues. Similar for those who have looked at these reports without drawing this inference.

If the pilots "worked", then it is likely because they were not real tests of what is claimed that they tested. I have not looked into this deeper than **Appendix H**, partly since I am still trying to understand what that confusing notion of MTA actually is.

Platform Wiskunde Nederland (PWN) as a historical throwback

The "Platform Wiskunde Nederland" (PWN) is a creation in 2010 of both the research mathematicians in "Koninklijk Wiskundig Genootschap" (KWG) and the teachers of mathematics in "Nederlandse Vereniging van Wiskundeleraren" (NVvW). [87]

- The teachers actually broke apart in 1925, since *education* is an empirical activity and research topic and thus quite different from *research in mathematics*.

[87] http://www.platformwiskunde.nl/over_pwn.htm

- Abstract thinking mathematician Hans Freudenthal (1905-1990) created RME now at FHCRMI in Utrecht. Results in mathematics education went down. FHCRMI could not spot the error in the KNAW report of 2009, [88] its ideologues do not acknowledge that RME failed, [89] and Koeno Gravemeijer even tries to use the "21st century skills" to support RME via a back door. [90]
- The creation of PWN can be understood from the "math war" in Holland. The research mathematicians in KWG did a power grab by creating PWN and linking NVvW closer to KWG.
- Apparently a key actor was Henk Broer, professor of mathematics in Groningen and head of the KNAW mathematics section. In the mid-1970s, Broer got his teaching permit, but never actually used it, and he did no research in mathematics education, but still considers himself competent to judge (while he isn't't). [91]
- PWN presented the Dutch minister of education with a misrepresenting report on math and math ed. [92]
- Apparently PWN wants to embellish its position, and some videos on the cTWO programme can be found at their website and not at the website of the teachers. Well, if a bureaucracy exists, then it must prove its "raison d'être". [93]

As an economist, my advice in 2008 was to create a "Simon Stevin Institute" (SSI) to resolve the power void in mathematics education. [94] [95] Nowadays this would also involve the abolition of PWN. Let KWG concentrate on mathematics for adults (like research mathematics). It is a dangerous confusion that mathematicians in TE would know about mathematics education in SE and PE. Obviously advice is welcome, but they cannot bear responsibility that they are not competent for. The major errors already made should flash alarm.

As an economist, my advice in 2008 was a parliamentarian enquiry into mathematics education and its research. Nowadays, I also propose that witnesses are heared under oath, and face criminal charges if they were to commit perjury.

Empirical research fields that use mathematics are equipped to both understand mathematics and respect empirics. They can recognise the difference between astronomy and astrology. They are advised to set up a (large) commission to analyse what has been happening here. Observe that they cannot hear witnesses under oath unless there would be criminal charges. My suggestion is that they closely co-operate with Parliament. The objective remains to find information for future lawmaking, while FHCRMI should not be at a university.

Interaction of exam programme, publishers, teachers

Let us consider the following conventions:

(1) Dutch teachers rely very much on textbooks. (This might have to do with workload, or with larger schools that want to avoid differences between classes, or with more assertive students who want to know why one deviates from the book that they already paid for.)
(2) Authors of textbooks closely watch the exam requirements.
(3) Researchers in didactics of mathematics teach future teachers of mathematics, and they seldomly retrain practicing teachers.

In this setting cTWO might have reasoned:

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[88] http://thomascool.eu/Papers/Math/CWI-Leiden/2016-09-30-Letter-to-CWI-anonimised.pdf
[89] https://boycottholland.wordpress.com/2016/07/11/pierre-van-hiele-and-michiel-doorman-doorman-misleads-indonesia-too
[90] http://www.wiskundebrief.nl/724.htm#6
[91] http://thomascool.eu/Papers/AardigeGetallen/2016-10-05-Afscheidsrede-Henk-Broer-en-diens-KNAW-sectie-voorzitterschap.pdf
[92] http://thomascool.eu/Papers/AardigeGetallen/2016-03-22-Minister-krijgt-een-misleidend-Deltaplan-Wiskunde.pdf
[93] http://www.platformwiskunde.nl/onderwijs_examenprogrammas_vanaf_2015.htm
[94] https://boycottholland.wordpress.com/2015/10/31/the-power-void-in-mathematics-education
[95] http://thomascool.eu/Papers/AardigeGetallen/2016-03-03-Het-Simon-Stevin-Instituut.pdf

> When we introduce MTA in the final exams, then the authors of the textbooks will adapt, and then the teachers will follow suit.

The issue of insight however is old – as cTWO acknowledges – and thus teachers will have heard about it in their own education as teacher. Pierre van Hiele's thesis of 1957 dates from almost 60 years ago, longer than any reasonable teaching career. Also, the documents about MTA only repeat the ideals that are already in the books on didactics – albeit not the specific approach towards implementation.

Thus the problem is not that the teachers do not know about those ideals.

There is no (serious) teacher who wants to teach students *only some tricks* to pass the next exam, which tricks they immediately forget after the exam. There are commercial bureaus who might do so. On occasion, a teacher might consider this a second-best, but this is no aim.

Why does cTWO write up what teachers already know, while explaining that it isn't new ?

A problem might be the textbook authors.

Perhaps textbook authors have a different view on didactics than cTWO ?

A solution is:

- Let textbooks be open access.
- Let there be forums where issues are discussed.
- Let there be open acces test questions and testing software.

This is however not what cTWO proposed.

The phrase MTA thus might have another origin.

Perhaps Anne van Streun really pushed for aiming at learning to think, and then the RME and TME ideologues jumped to the opportunity, each seeing something worthy for themselves (RME the use of contexts, TME the proving), and nobody maintained a critical stand, that "learning to think" should not be tested in the final exam.

Still, the exam regulation is a wrong platform for such an objective.

A consequence might be that now the proper approach to reach that objective will not be used. The proper approach is improvement in didactics. And above suggestion on open access.

Selective references, also to references that fail w.r.t. scientific integrity

While the programme with MTA was finalised in 2012, its creators and new associates continue developing supportive material. For this, they may use selective references (from their own "club").

Anne van Streun repetitively presents his bicycling case but with no discussion about what it would mean with the framework of Van Hiele's analysis. [96]

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[96] Anne van Streun, (i) inaugural lecture, "Het denken bevorderen", NAW http://www.nieuwarchief.nl/serie5/pdf/naw5-2002-03-4-294.pdf, (ii) Chapter 1 "Leren en onderwijzen van wiskunde", "Handboek Wiskundedidactiek" (2012) p15 bicycle and p350 walking, or (iii) the similar problem on p61 of "Ontwerpen van wiskundige denkactiviteiten bovenbouw havo-vwo", http://downloads.slo.nl/Repository/ontwerpen-van-wiskundige-denkactiviteiten-bovenbouw-havo-vwo.pdf, (iv) and another time that I recall from somewhere. (v) PM. Van Streun's thesis is not online yet, only a summary: http://www.rug.nl/research/portal/publications/heuristisch-wiskundeonderwijs-verslag-van-een-onderwijsexperiment(664f4779-79a2-43d8-b54e-94318d5b2236).html

Anne van Streun & Peter Kop (2016), "Ontwerpen van wiskundige denkactiviteiten bovenbouw havo-vwo", [97] SLO, p8, refer to these two references that fail against integrity of science:

- Drijvers, Van Streun & Zwaneveld (2012), "Handboek Wiskundedidactiek", Epsilon Uitgaven
- Roorda (2012), "Ontwikkeling in verandering," Thesis RUG, Groningen

Namely, I observed that Roorda did not refer to my algebraic approach to the derivative, even though I informed him about this in 2008. He does this neither in his thesis on teaching the derivative nor in the overview article in the "Handboek" with the same topic. I consider this a breach of integrity of science. I have asked the editors of the "Handboek" to correct the situation. However, the "Handboek" now is running at a 4th unchanged edition. Both researchers and students training to become teachers of mathematics in Holland are not getting the relevant information.

I developed the approach in 2007. A reasonable period for implementation for actual use in education would be 10 years. We are close to 2017 and still quite far from such an implementation. (A reader might think: we have only this author's view about the usefulness of the approach. This, however, is where part of the beauty of mathematics lies: you can check for yourself. The PDF of "Conquest of the Plane" (COTP) (2011) is online. [98] See also the reading notes. Notice that any competent researcher in the didactics of mathematics could have made the steps from 2007 to 2011 with respect to the algebraic approach to the derivative. There obviously are elements of creativity for other aspects that remain with an individual author. For example, the decision that it is better to start with area and integral and end up with slope and derivative, requires a sensitivity to didactics that some authors may lack. All of this of course still must be tested, since it are students who tell us what works.)

The situation has become rather messy:

- Appeal to President of KNAW: http://thomascool.eu/Thomas/Nederlands/Wetenschap/Brieven/2012-06-09-AanPresidentKNAW.html
- Collection of letters: http://thomascool.eu/Papers/BHRM/2015-10-28-Malconduct-Roorda-Daemen-Drijvers.pdf
- Question to Epsilon Uitgaven whether they are a scientific publisher, which they claim, while Drijvers argues that the "Handboek" is no scientific publication: http://thomascool.eu/Papers/AardigeGetallen/2015-10-30-Brief-aan-redactie-Epsilon-Uitgaven.html

Conclusions

There are more, but the abstract gives the main conclusion.

[97] http://downloads.slo.nl/Repository/ontwerpen-van-wiskundige-denkactiviteiten-bovenbouw-havo-vwo.pdf
[98] http://thomascool.eu/Papers/COTP/Index.html

Appendix A. MTA and "realistic contexts" in the Dutch regulation

The Dutch regulation on VWO (pre-university) examination in 2018 (and also the pilot schools in 2017) has these points on MTA and "realistic problems" and "contexts". [99] [100]

In itself, there is nothing wrong with "realistic problems", when they are taken as "problems derived from reality". Also the subject matter of Euclid can be regarded as such. In itself there is nothing wrong with the word "context" too.

There would be a problem however if the regulation would be interpreted as making *the didactics of RME mandatory.* This namely is not the objective of an exam regulation.

Page 10 (my emphasis in bold):

> **Subdomein A2: Profielspecifieke vaardigheden**
> 2. De kandidaat kan profielspecifieke probleemsituaties in wiskundige termen analyseren, oplossen en het resultaat naar het oorspronkelijke probleem terugvertalen.
>
> De kandidaat kan
>
> > 1. een probleemsituatie in een wiskundige, natuurwetenschappelijke of maatschappelijke context analyseren, gebruik makend van relevante begrippen en theorie vertalen in een vakspecifiek onderzoek, dat onderzoek uitvoeren, en uit de onderzoeksresultaten conclusies trekken;
> > 2. een **realistisch probleem in een context** analyseren, inperken tot een hanteerbaar probleem, vertalen naar een wiskundig model, modeluitkomsten genereren en interpreteren en het model toetsen en beoordelen;
> > 3. met gegevens van wiskundige en natuurwetenschappelijke aard consistente redeneringen opzetten.
>
> **Subdomein A3: Wiskundige vaardigheden**
> 3. De kandidaat beheerst de bij het examenprogramma passende **wiskundige denkactiviteiten** – waaronder modelleren en algebraïseren, ordenen en structureren, analytisch denken en probleemoplossen, formules manipuleren, abstraheren, en logisch redeneren en bewijzen - en kan daarbij ICT functioneel gebruiken.

Appendix B. Some other sources on "Mathematical Think Activities"

This appendix mentions some other resources on "Mathematical Think-Activities" (MTA).

An overview article in Euclides is in:
http://www.fi.uu.nl/ctwo/publicaties/docs/Euclides825(169-172).pdf

The NVvW annual convention 2015 (and surprisingly hardly in 2016):
https://www.nvvw.nl/21753/jaarvergadering/jaarvergadering-2015

"Netherlands Initiative for Education research", NWO-projectnummer: 405-14-502:
https://www.nro.nl/kb/405-14-502-wiskundige-denkactiviteit-in-wiskunde-op-havo-en-vwo
Observe that the project at Maurick college concerned the quadratic function and quadratic formula. [101] (**Appendix G.**) However, it is important to know what is the teaching objective.

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[99] https://www.examenblad.nl/examenstof/syllabus-2018-wiskunde-b-vwo/2018/f=/syllabus_wiskunde_b_vwo_2018.pdf
[100] I was alerted to this by Drijvers, sheet 15: http://www.ru.nl/publish/pages/779511/2016-05-12leonvandenbroeklezingnijmegenwda-drijvers.pdf
[101] https://www.youtube.com/watch?v=CbOmcsbOwGE&feature=youtu.be

TME and RME are focused on the polynomial, and have only different didactics. For a neoclassical reengineering (NME) see [102] or Dutch [103].

A database of questions: http://www.fisme.science.uu.nl/publicaties/subsets/wda

With references in English: https://www.leraar24.nl/dossier/6100#tab=0

Such a reference is to Schoenfeld 1992: http://howtosolveit.pbworks.com/f/Schoenfeld_1992%20Learning%20to%20Think%20Mathematically.pdf

Ben Wilbrink refers to "Peak: Secrets from The New Science of Expertise. By Anders Ericsson and Robert Pool, Bodley Head, 336pp, £18.99, ISBN 9781847923196, Published 21 April 2016: https://www.timeshighereducation.com/books/reviews-peak-secrets-from-the-new-science-of-expertise-anders-ericsson-robert-pool-bodley-head

Apparently, Wilbrink has no separate page on MTA / WDA (yet): http://benwilbrink.nl/literature/handboek_wiskundedidactiek.htm

Joost Hulshof has a fairly incoherent column, but a key line is: "Die wiskundige denkactiviteiten (WDA), zijn die te leren of toetsen? Anne van Streun denkt dus van wel. Ik niet, althans niet wat toetsen betreft." http://www.beteronderwijsnederland.nl/content/nieuwe-examenprogrammas-wiskunde-het-onderwijzen-en-toetsen-van-wiskundige-denkactiviteiten

There are at least two references at this session: http://www.ru.nl/pucofscience/docenten-schoolleiders/activiteiten/studiedagen/wiskunde/wiskundedialoog-2016

- Test designer Ger Limpens doesn't show how he scores his test questions: http://www.ru.nl/publish/pages/779511/2016_05_nijmegen_wda_en_toetsing_ger_limpkens.pdf
- Paul Drijvers refers to
  (a) Keith Devlin, who is known as mathematician and populiser of mathematics, but not as an *empirical researcher* in mathematics education. http://www.ru.nl/publish/pages/779511/2016-05-12leonvandenbroeklezingnijmegenwda-drijvers.pdf My own deconstruction of some arguments by Kevlin: https://boycottholland.wordpress.com/2014/06/03/math-philosophy-for-a-general-audience
  (b) a master's thesis by Hanneke Kodde-Buitenhuis (2016) (of which he doesn't state on the sheet that he was the supervisor) that shows that the regular exams already contain MTA, and that the new exams only contain more of those. Apparently it depends upon what one calls "MTA".
  (c) His triangle that I find confusing.

Drijvers potentially has publications on the topic, but I have not checked. [104] He published two articles in Euclides, the journal of Dutch NVvW. Apparently the editors of the journal thought that these articles were fit to print, and did not think that the articles were confused on the notion of MTA. Euclides however is not a scientific journal. (Drijvers indeed does not list these articles under his "scientific publications". One should beware however that such "scientific publications" about RME would well be ideology comparable to astrology or homeopathy. Also, Drijvers lists "Handboek Wiskundedidactiek" not under the "scienctific publications", but it has been published by Epsilon Uitgaven under its scientific label. [105])

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[102] https://boycottholland.wordpress.com/2016/04/24/teaching-quadratic-functions-re-engineered
[103] http://www.wiskundebrief.nl/738.htm#5
[104] http://www.uu.nl/medewerkers/phmdrijvers/0
[105] http://thomascool.eu/Papers/AardigeGetallen/2015-10-30-Brief-aan-redactie-Epsilon-Uitgaven.html

Appendix C. The experts who made the cTWO report, some footnotes

Some general principles

Ad hominem is: John is a farmer and he shouldn't speak about paintings.

Judging on expertise is: Paula is a professor in mathematics and is not qualified for secondary education in mathematics and its research. Paula may ask useful questions but you would not hire her as an expert because there are others who have the proper qualification.

Note the order of reasoning. Valid is: Wilma made this error, and thus she can't really be an expert. Ad hominem is, unless supported by judgements on expertise: Wilma is not an expert and thus she will make errors like this.

Query about the professional position of Anne van Streun

Anne van Streun has no webpage or online cv. His LinkedIn page doesn't provide details, and it is only an educated guess that it must be him because of his connections. [106] Van Streun got a professorship late in life, in 2002, [107] and it is stated that he started as a teacher in 1964 and became a teacher trainer in 1974. He must have been born in 1940-1944, since he might already have started teaching before graduation. When I studied in Groningen around 1980 it was already known that you might do a course of about three months with him for a qualification for mathematics teacher. At cTWO around 2010 he might be 66-70 years of age, and in 2016 he might be around 72-76 years of age. A short cv might be in the thesis of 1989, but this is not online. [108] It is not clear what the relationship of his thinking would be to Van Hiele and Freudenthal (except for page 297 of the lecture in NAW). I wonder whether he has ever noted Freudenthal's abuse of Van Hiele's work (it would be in his field of expertise). [109]

Two key points of the thesis on "heuristic mathematics education" (HME): [110] (i) students learn to better *apply mathematics* to solve problems, (ii) test problems are taken from *applied mathematics.*

From the summary: HME is compared to "First mathematics, then applications" (FMTA) [111] and "HEWET". [112] There are a "tight experimental core" of 5 classes and 2 teachers and in a "broad group" of 21 classes and 16 teachers. The results of testing:

- Applying mathematics for applied questions: HME > HEWET > FMTA
- Command of basic concepts and techniques: HME > FMTA > HEWET
- Better command of mathematics causes less use of heuristics and more use of algorithms.

There might be a confusion between the learning paths in Table 1.

[106] https://www.linkedin.com/in/anne-van-streun-10905933
[107] http://www.nieuwarchief.nl/serie5/pdf/naw5-2002-03-4-294.pdf I have not deconstructed this yet.
[108] http://www.rug.nl/research/portal/publications/heuristisch-wiskundeonderwijs-verslag-van-een-onderwijsexperiment(664f4779-79a2-43d8-b54e-94318d5b2236).html
[109] http://www.wiskundebrief.nl/718.htm#7
[110] http://www.rug.nl/research/portal/files/14472200/heuristisch_wiskunde.pdf!null
[111] In 1989 apparently done by the textbooks "Getal en Ruimte" and "Sigma".
[112] Experimental programme 1981-1985. See e.g.: http://www.fisme.science.uu.nl/publicaties/subsets/hewet Though apparently also included in the textbook "Moderne Wiskunde".

The authors of cTWO

The vision document *"Rijk aan betekenis"* (2007) has these authors (p62).

PM. I was member of the NVvW working groep of HBO (colleges), and know that Roel van Asselt and Jan Blankespoor are competent colleagues, though I don't know their research on mathematics education. Some authors are unknown to me. For the others, I can incude footnotes.

- Roel van Asselt, Saxion Hogeschool Enschede / LICA
- Frits Beukers, hoogleraar wiskunde UU [113]
- Jan Blankespoor, leraar hbo
- Henk Broer, hoogleraar wiskunde RUG, voorzitter kamer Wiskunde VSNU [114]
- Paul Drijvers (secretaris), onderwijsontwikkelaar Freudenthal Instituut [115]
- Swier Garst, leraar wiskunde [116]
- Carel van de Giessen, leraar wiskunde
- Rainer Kaenders, vakdidacticus RUN [117]
- Wim Kleijne, oud-inspecteur onderwijs, lid van beide profielcommissies
- Marian Kollenveld, leraar wiskunde en voorzitter NVvW [118]
- Mark Peletier, hoogleraar wiskunde TUE [119]
- Dirk Siersma (voorzitter), hoogleraar wiskunde Universiteit Utrecht, voorzitter NOCW [120]
- Anne van Streun, emeritus hoogleraar en vakdidacticus RUG
- Chris Zaal, docent wiskunde-educatie en onderwijsontwikkelaar UvA [121]

[113] Mathematician with no qualification for SE. Got funds for NRO "research" while he is not qualified for education research. Also participated in slandering against didactic innovation on Θ = 2π. See p26-29 http://thomascool.eu/Papers/AardigeGetallen/2016-07-04-Nav-JanHogendijk-en-DonQuichot.pdf

[114] Prime ideologue of TME in the "math war", see http://thomascool.eu/Papers/AardigeGetallen/2016-10-05-Afscheidsrede-Henk-Broer-en-diens-KNAW-sectie-voorzitterschap.pdf

[115] Claims that "Handboek Wiskundedidactiek" (2012) is no scientific book but published this in the science series of Epsilon Uitgaven and includes it on his list of relevant publications. Allows Gerrit Roorda and Joke Daemen to misrepresent the situation about research on didactics of the derivative. See http://thomascool.eu/Papers/BHRM/2015-10-28-Malconduct-Roorda-Daemen-Drijvers.pdf

[116] Since December 2014 chairman of NVvW, see the "red card" in June 2016: http://thomascool.eu/Papers/Math/2016-06-28-Letter-to-NVVW-with-Red-Card.pdf

[117] I gave him a copy of "Elegance with Substance" (2009) and did not receive a reaction. Kaenders had an important interview with Pierre van Hiele, in NAW 2005, see https://boycottholland.wordpress.com/2015/10/11/pierre-van-hiele-and-an-interview-in-2005. Unfortunately, Kaenders doesn't seem to understand that Hans Freudenthal really abused the work by Van Hiele, see https://boycottholland.wordpress.com/2015/09/26/pierre-van-hiele-and-the-history-of-mathematics

[118] Helped create PWN, which is a throwback of history. Chief responsitble for allowing misrepresenting and abusive "book reviews" in Euclides about "Elegance with Substance" (2009) and "Conquest of the Plane" (2011). My diagnosis was that NVvW is a seriously sick organisation. http://thomascool.eu/Papers/AardigeGetallen/2012-06-Gedoe-bij-Euclides.html

[119] Research mathematician without qualification for SE or its research. Secretary of KWG in 1998-2003. Commited censorship on the "Mathematics Press Service" so that my diagnosis is that this is a "Mathematics Lie and Deception Service", http://thomascool.eu/Thomas/Nederlands/Publiek/Artikelen/2007-04-14-WPD.pdf Participated in slandering at the BON Forum (check the issue with Beukers): http://www.beteronderwijsnederland.nl/content/sinus-en-radialen-te-moeilijk-voor-kinderen-laten-we-iets-nieuws-bedenken and http://thomascool.eu/Thomas/Nederlands/Wetenschap/Brieven/2009-02-19-BON-Gedoe.pdf

[120] He signs the cTWO report to the minister with his titles "prof. dr." but these titles concern research mathematics, and his should have stated "amateur", since he is not qualified for SE or research in mathematics education. NOCW became later part of PWN, a throwback of history. NOCW has been part of IMU / ICME, with Siersma as its representative, see https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics

[121] https://www.linkedin.com/in/cgzaal I have no experience with Zaal. He got his teaching degree but after one year of teaching switch to TU Delft. He was editor and did one year of teaching again, and joined FHCRMI for three years, then getting a Ph.D. in 2005 in research mathematics, which is his

The cTWO final report of 2012 adds these persons.

PM. Peter Kop has been my teacher trainer at ICLON Leiden, and I am full of praise for his approach to teaching. I have not looked into his (recent) research. Christiaan Boudri was member of the NVvW working group on HBO and has taken a laudable public stand for reason and keeping an open mind. [122]

- Marja Bos, leraar wiskunde en schooldecaan (per 1-1-2010) [123]
- Christiaan Boudri, docent wiskunde en toepassingsvakken Hogeschool Arnhem Nijmegen (per 1-1-2010)
- Peter Kop, leraar wiskunde, vakdidacticus bij Iclon, Universiteit Leiden (per 1-1-2010)
- Ronald Meester, hoogleraar wiskunde Vrije Universiteit (per 1-1-2010) [124]
- Jeroen Spandaw, universitair docent en vakdidacticus wiskunde TU Delft (per 1-1-2010) [125]

The final report also mentions a project team:

- Peter van Wijk (teamleider per 1-8-2009)
- Hielke Peereboom
- Theo van den Bogaart (secretaris cTWO per 1-1-2011) [126]
- Sieb Kemme (teamleider tot 1-8-2009)
- Ivo Claus (tot 1-8-2008)
- Swier Garst (projectuitvoering Wiskunde D van 1-3-2007 tot 1-8-2007) [127]
- Pauline Vos (projectuitvoering Wiskunde D tot 1-3-2007) [128]

In the working group of collaboration between physics and mathematics, there is Henk van der Kooij of FHCRMI. [129] In the commission on "math D" there is Michiel Doorman of FHCRMI who in 2016 still claims that RME would be better even though there is the KNAW report of 2009. [130]

And as one of the employees of other organisations that were involved intensively, one mentions on Ger Limpens (page 235) of CITO, who in 2010 gave an ambiguous "book review" of "Elegance with Substance". He really finds it useful to pose the question whether I would be a wierdo ("zonderling", the translation as "eccentric" does not have the Dutch

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continued position.
http://dare.uva.nl/search?field1=keyword;value1=Zaal%202005;docsPerPage=1;startDoc=1

[122] http://thomascool.eu/Papers/COTP/2013-03-15-Boudri-over-COTP.pdf

[123] She was editor of Euclides before the slanderous "book reviews" of EWS and COTP were published. My impression is that she did an overall good job. Unfortunately, she declined interest in reviewing "A logic of exceptions" (ALOE) and "Voting theory for democracy" (VTFD), arguing that these subjects would not be interesting for teachers of mathematics. ALOE also presented the algebraic approach to the derivative. http://thomascool.eu/Papers/AardigeGetallen/2007-02-26-ALOE-en-VTFD-afgewezen-door-Euclides.html Perhaps a decent review of these books in 2007 would have meant a world of difference.

[124] https://www.linkedin.com/in/ronald-meester-38b71b115 As far as I can see a research mathematician without qualification for SE or its research.

[125] Two Ph.D. titles. Teacher trainer at TU Delft. Wrote a misrepresenting and slanderous "book review" in Euclides 2012 about "Conquest of the Plane" (COTP) (2011). The TU Delft commission on scientific integrity failed to set up a reading commission, and decided without reading that the issue was not important enough to look into and that my protest could be neglected as if I would be a person who would protest without proper cause: http://thomascool.eu/Papers/COTP/LOWI/Index.html

[126] Rejected, as secretary, that I would present EWS and COTP to cTWO, see http://thomascool.eu/Papers/Math/2013-02-06-Colignatus-nav-cTWO-Eindrapport.html

[127] See above.

[128] Presented a lecture at the NVvW annual convention in 2015 in which she referred to work by Gerrit Roorda but did not mention the breach in integrity of science.

[129] http://thomascool.eu/Papers/AardigeGetallen/2016-03-11-Henk-vd-Kooij-Freudenthal-VanHiele.pdf

[130] https://boycottholland.wordpress.com/2016/07/11/pierre-van-hiele-and-michiel-doorman-doorman-misleads-indonesia-too

flavour) and he refers to Don Quixote, but in the end states that the book has its use. [131] PM. CITO also provides tests for PE on arithmetic but failed to see the conceptual error in the KNAW 2009 report on arithmetic. [132]

Appendix D. A curious petition by Anne van Streun in 2008

In 2008, the minister of Education accepted apparently much of the cTWO proposals on "mathematics B" but went along much with the Resonansgroep on "mathematics A".

- Anne van Streun started an internet petition "Stop destruction of mathematics education" [133] This got some 500 signatures, including from Marian Kollenveld (chair NVvW) and Jan van Maanen (director of Freudenthal Head in the Clouds Realistic Mathematics Institute). Apparently ipetitions doesn't show the signatories.
- Jan van de Craats (2009:31) judged this petition "misleading to the non-experts". [134] [135] See also [136].

Text on ipetitions:

"open brief STOP KAALSLAG WISKUNDEONDERWIJS Geachte mevrouw van Bijsterveldt, In overgrote meerderheid neemt de Tweede Kamer zich voor om in de toekomst niet meer eenzijdig politieke beslissingen te nemen waar in het onderwijsveld geen draagvlak voor is. Tegelijkertijd besluit u, Staatssecretaris van Bijsterveldt, om de voorgestelde examenprogramma\'s wiskunde havo-vwo zo sterk eenzijdig te amenderen dat van de nagestreefde evenwichtige balans tussen de verschillende dimensies van de wiskunde en het wiskundeonderwijs niets meer is overgebleven. Onbegrijpelijk omdat de voorstellen zijn opgesteld door de vernieuwingscommissie wiskunde cTWO, die door minister Van der Hoeven is samengesteld en met deze taak is belast. Onbegrijpelijk omdat deze commissie breed is samengesteld met onder andere vijf hoogleraren, vertegenwoordigers van het hbo, en een vertegenwoordiging van de leraren in havo-vwo door de Nederlandse Vereniging van Wiskundeleraren. Onder leiding van cTWO hebben tientallen wiskundeleraren, vertegenwoordigers van hbo/wo en diverse medewerkers van de verschillende onderwijsinstituten in een tweetal jaren evenwichtige examenprogramma\'s ontwikkeld voor de vakken wiskunde A, B, D (havo en vwo) en C (vwo). Evenwichtig niet alleen ten aanzien van de doorstroomrelevantie maar daarnaast ook meewegend de diverse algemeen vormende en culturele aspecten van wiskunde en wiskundeonderwijs. Verbijsterend is de didactische toelichting bij uw besluit. Een toelichting die een internationaal volstrekt unieke beschouwing bevat over wat de kern van de wiskunde in het algemeen vormend wiskundeonderwijs behoort te zijn. Die kern is volgens die toelichting het algebra [and here the internet text stops abruptly]"

PDF text by ANP. [137]

[131] http://thomascool.eu/Papers/Math/2010-12-Euclides-86-3-p130-131-a.jpg and http://thomascool.eu/Papers/AardigeGetallen/2010-12-23-AardigeGetallen-p169-171-Bespreking-door-Limpens-deugt-niet.pdf
[132] http://thomascool.eu/Papers/Math/CWI-Leiden/2016-09-30-Letter-to-CWI-anonimised.pdf
[133] http://www.ipetitions.com/petition/wiskunde
[134] http://www.nieuwarchief.nl/serie5/pdf/naw5-2009-10-1-029.pdf
[135] https://staff.science.uva.nl/j.vandecraats/Kanttekeningen_open_brief.pdf
[136] http://oud.digischool.nl/wi/WiskundE-brief/458.htm#2
[137] https://www.parlementairemonitor.nl/9353000/1/j9vvij5epmj1ey0/vhvyf7ggy9v7

Appendix E. Copy of "Bijlage 11 De grafische rekenmachine (GRM)"

[Copied from http://downloads.slo.nl/Repository/afstemming-wiskunde-natuurkunde-tweede-fase.pdf page 67-68. Observe that the focus on the use of the GC comes at the cost of developing computer algebra, since 1995.]

De verschillen tussen de grafische rekenmachine (GRM) en de wetenschappelijke rekenmachine liggen in de eerste instantie op het gebied van functieonderzoek:

- plotten van grafieken;
- numerieke bepaling van coördinaten van bijzondere punten van grafieken (toppen,
- snijpunten met de x -as, snijpunten van twee grafieken);
- numerieke bepaling van de afgeleide;
- numerieke bepaling van integralen.

In tweede instantie heeft de GRM uitgebreide functies op het gebied van kansrekening en statistiek:

- Gegevens kunnen worden ingevoerd in een tabel en vervolgens geanalyseerd met statistische functies (variantie, standaardafwijking, mediaan, kwartielen).
- Combinatorische functies en kansverdelingsfuncties zijn beter toegankelijk; in het bijzonder is het vaak niet meer nodig om kansen te berekenen via de standaard normale verdeling.

Wetenschappelijke rekenmachines beschikken ook over een deel van deze functies, maar met de GRM gaat alles veel overzichtelijker en gemakkelijker.

De grafische component van de GRM opent echter op het gebied van kansrekening ook geheel nieuwe mogelijkheden, zo zijn kansverdelingsfuncties te plotten als functie van een invoervariabele.

Een integraal over een normale verdeling kan bijvoorbeeld naar believen worden geplot als functie van de ondergrens, de bovengrens, de verwachtingswaarde μ of de standaardafwijking σ .
Dat brengt een nieuwe manier mee om bepaalde opgaven op te lossen. Als in een opgave over de normale verdeling bijvoorbeeld wordt gevraagd om μ te berekenen, kan een leerling de integraal over de normale verdeling plotten als functie van μ , en de GRM vervolgens de grafiek laten snijden met een horizontale lijn die de gewenste kans vertegenwoordigt.

Op een vergelijkbare manier kan een leerling moeizaam gebruik van arcsin, arccos en arctan omzeilen. Bij een opgave als “bereken α in drie decimalen nauwkeurig als sin α = 2/5 waarbij ½ $\pi < \alpha < \pi$ ” hoeft de leerling slechts de grafieken van y = sin x en y = 2/5 te plotten en het juiste snijpunt te bepalen. [138]

De GRM heeft nog veel meer mogelijkheden die we niet allemaal de revue laten passeren. Het is duidelijk dat de verschillen met een standaard wetenschappelijke rekenmachine zeer groot zijn, en dat deze verschillen zich vooral bij wiskunde manifesteren.

De syllabuscommissies wiskunde (A, B en C) hebben er voor gepleit om de GRM toe te laten. Zij noemen daarbij argumenten als:

- Het gebruik van ICT is in de examenprogramma's een integraal onderdeel. Dat gebruik van ICT is voor de wiskunde uitvoerbaar door het gebruik van de GRM toe te staan.
- In de centrale examens in de pilots is de GRM noodzakelijk . Het onderwijs is ook volledig op het altijd aanwezig zijn van de GRM afgestemd.
- De introductie en uitwerking van de in genoemd subdomein A3, punt 1 beschreven wiskundige denkactiviteiten is een belangrijk aspect van de vernieuwing van de

[138] Comment by TC 2016-10-29: The point {x, y} = {cos[phi], sin[phi]} is defined for the unit circle. Thus the use of y = sin[x] is hopelessly confused.

examenprogramma's wiskunde. Leerlingen leren daarbij te kiezen tussen een berekening of benadering, het aflezen van een tabel of van een grafiek, het krijgen van een indruk van het verloop van een grafiek, het voorspellen en controleren van antwoorden, het dynamiseren van wiskundige modellen door het variëren van de optredende parameters. De GRM is een essentieel onderdeel van deze vaardigheden. Hier hoort ook bij het inzicht in de beperkingen van de GRM bij het maken van numerieke benaderingen.
- De GRM heeft een duidelijke meerwaarde op twee belangrijke terreinen:
    - bij het oplossen van authentieke problemen, d.w.z. problemen uit de dagelijkse praktijk waar het eerder uitzondering dan regel is dat je mooie getallen hebt, dan wel mooie uitkomsten
    - bij het werken met parametervoorstellingen van grafieken, waarbij de dynamiek van de GRM benut wordt.
- De GRM is een normaal en frequent gebruikt hulpmiddel in alle lessen wiskunde in de bovenbouw havo/vwo. Veel leerlingen ontlenen aan de grafische rekenmachine een zekere mate van examenzekerheid.
- De GRM is niet alleen een hulpmiddel om te rekenen of te tekenen, maar is ook een didactisch gereedschap om concepten in te voeren en te verduidelijken.
- De GRM speelt een essentiële tijdbesparende rol. Dat geldt zeker waar het om het werken met realistische wiskundige modellen gaat, die immers altijd benaderende waarden bevatten.

Een deel van deze argumenten geldt ook voor natuurkunde. Ook daar is de GRM een normaal en frequent hulpmiddel en wordt deze als didactisch gereedschap gebruikt.

Appendix F. Neoclassical re-engineering of teaching the quadratic function

The following was included in the mathematics teaching newsletter. [139] The discussion in English is here. [140] [141] One can compare this neoclassical re-engineering with the MTA approach to a lesson on the topic, **Appendix G**.

Title: Een betere didactiek voor kwadratische functies

Neem de kwadratische functie $f(x) = x^2$, vervorm die met coëfficiënt a en verplaats deze h stappen horizontaal en v stappen verticaal. Je krijgt dan de functie $f(x) = a(x - h)^2 + v$. Uit deze basisvorm is het draaipunt (h, v) van de parabool direct af te lezen. Ook de nulpunten volgen eenvoudig. Dit is grafisch allemaal direct te tonen.

Het bovenstaande vormt een veel betere didactiek voor kwadratische functies dan de traditionele aanpak die uitgaat van het polynoom $f(x) = ax^2 + bx + c$. Voor het leerproces is deze polynoom als startpunt irrelevant en werkt deze juist antididactisch.

In de vorm $f(x) = a(x - h)^2 + v$ rekt, krimpt of spiegelt parameter a. Met $h > 0$ gaat de grafiek naar rechts, met $h < 0$ naar links. Met $v > 0$ gaat de grafiek omhoog, met $v < 0$ omlaag. Ik gebruik verder liever de term "draaipunt" dan "top", want wat is de "top" van een dalparabool?

Abstractie contra empirie

Wiskundigen zijn getraind tot abstract denken terwijl onderwijs een empirische zaak is. Wanneer je die twee zaken door elkaar haalt, geeft dat in het onderwijs veel ellende. Dit heb ik met veel voorbeelden toegelicht in mijn boek "Elegance with Substance" (2009, 2015). Onlangs zag ik in dat ook de kwadratische functie onder die misplaatste abstractie lijdt.

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[139] http://www.wiskundebrief.nl/738.htm#5
[140] https://boycottholland.wordpress.com/2016/04/24/teaching-quadratic-functions-re-engineered
[141] https://boycottholland.wordpress.com/2016/05/01/a-long-road-with-a-recipe

Waar de traditionele aanpak van de kwadratische functie vandaan komt, mogen historici uitzoeken. Is het een residu van vroege ontdekkingen rond 1500 voor Christus? Of komt het inderdaad voort uit de abstracte wiskundige theorie rond polynomen? Voor leerlingen is het polynoom in ieder geval niet inzichtelijk. De "basisvorm" $f(x) = a(x - h)^2 + v$ biedt leerlingen daarentegen meteen een mooi inzicht. Door het kwadraat uit te werken, kan het polynoom daarna eenvoudig worden gevonden. De parameters *h* en *v* kunnen dan direct worden uitgedrukt in termen van *a*, b en c. Met deze relaties kunnen andersom *a*, b en c weer worden uitgedrukt in *a*, *h* en *v*.

Liever inzicht dan rekentrucs

Wanneer factoren niet meteen gezien worden, geeft de abc-formule de nulpunten van de kwadratische polynoomvorm en minimaliseert hierbij de rekenstappen. De abc-formule is handig maar het is een didactische inversie om het algemene geval vanaf het begin centraal te stellen. De focus van het onderwijs kan beter liggen bij inzicht dan bij zo'n inverse rekentruc. Pas wanneer het inzicht bestaat over wat zo'n parabool nu is, dan ontstaat de vervolgstap om bij een gegeven polynoom de nulpunten te vinden en ontstaat de belangstelling voor de abc-formule.

Wanneer je binnen het onderwijs kwadratische functies snel inzichtelijk kunt maken, ontstaat er ook ruimte voor het behandelen van de complexe oplossingen van kwadratische vergelijkingen. Dat is geen gekke gedachte; kinderen zijn bij verantwoorde didactiek tot verrassend veel in staat. Pierre van Hiele (1909-2010) stelde bijvoorbeeld voor al op de basisschool met vectoren te beginnen.

Praktijkstudies

Hoe laat je kinderen het beste kennis maken met wiskunde? Hoe bied je leerlingen de beste wiskundige inzichten? Het zijn de leerlingen die bepalen wat werkt en dat valt alleen te bepalen met behulp van praktijkstudies. Daar moeten dan wel middelen voor vrijgemaakt worden. Daarom pleit ik sinds 2008 voor een parlementair onderzoek met betrekking tot het wiskundeonderwijs.

Thomas Colignatus
Econometrist (Groningen 1982) en leraar wiskunde (Leiden 2008), Scheveningen

Ik adviseer u deze links:

Re-engineering van didactiek van kwadratische functies. [142]
A long road with a recipe. [143]
Re-engineering van wortel en complex getal. [144]
Wansink over complexe getallen, Euclides 51e jaargang no 4. [145]
Conquest of the Plane (2011). [146]
Mijn brief aan het Nationaal Regie-orgaan Onderwijsonderzoek (NRO). [147]
Mijn advies tot een parlementair onderzoek naar het onderwijs in wiskunde. [148]

[142] https://boycottholland.wordpress.com/2016/04/24/teaching-quadratic-functions-re-engineered/
[143] https://boycottholland.wordpress.com/2016/05/01/a-long-road-with-a-recipe
[144] https://boycottholland.wordpress.com/2016/04/26/teaching-square-root-and-complex-number-re-engineered
[145] http://vakbladeuclides.nl/archief/pdf/51_1975-76_04.pdf
[146] http://thomascool.eu/Papers/COTP/Index.html
[147] http://thomascool.eu/Papers/Math/2016-04-15-Letter-to-NRO.pdf
[148] http://www.ipetitions.com/petition/tk-onderzoek-wiskundeonderwijs

Appendix G.1. Copy of an example MTA lesson on quadratic forms

Copied from: http://www.fisme.science.uu.nl/toepassingen/28403

NB 1. This is only a lesson and no test on it.
NB 2. In the original the variables like *x* are not written in italics !
NB 3. **Appendix G.2** contains questions to Dirk Boleij, the teacher who performed this MTA lesson, and his answers. I thank him for permission to reprint these. It appears that he first presented his students with his own classification scheme, that he calls "WOKA-NOVA". Thus we should be careful in comparing results over different implementations.

Compare this with the neoclassical reengineering (NME) [149] (Dutch, [150] also in **Appendix F**.)

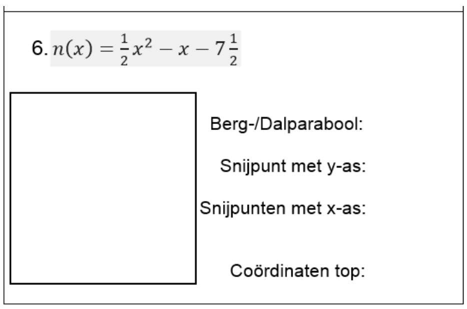
KWADRATISCHE VORMEN

Klas: 3 VWO
Onderwerp:
Kwadratische vormen
Leerdoel:
Herkennen van kwadratische vormen en eigenschappen hierbij.
Omschrijving:
Leerlingen leren hoe verschillende eigenschappen van grafieken direct af te lezen zijn bij bepaalde vormen van kwadratische functies. Tevens leren ze een geschikte vorm te kiezen bij een bepaalde grafiek.
Past bij:
Moderne Wiskunde , Editie 10, Hoofdstuk 11
Een docent heeft deze opdracht getest in de klas. Hiervan zijn filmopnamen gemaakt. Een filmpje van 10 minuten geeft een indruk van hoe deze les is verlopen. Daarnaast geeft de docent algemene tips voor het onderwijzen van wiskundig denken.
https://youtu.be/CbOmcsbOwGE

Opdracht

In de bijlage staan 12 kaartjes met verschillende functievoorschriften in verschillende vormen.

Maak met een computerprogramma een snelle grafiek van de gegeven functie en lees de bijzondere punten (zie kaartjes) af.
De uitwerkingen kun je op de kaartjes schrijven,
maar ook op het losse blad wat je erbij krijgt.
De vraag die gesteld wordt is: ‘wat valt op?’.



Uitwerking:

	Functie	Berg- dalparabool	Top	Snijpunt x-as	Snijpunt y-as
1	$f(x) = -(x+2)^2 + 16$	Berg	(-2,16)	(2,0) (-6,0)	(0,12)

[149] https://boycottholland.wordpress.com/2016/04/24/teaching-quadratic-functions-re-engineered
[150] http://www.wiskundebrief.nl/738.htm#5

2	h(x) = -2(x-1)(x-5)	Berg	(3,8)	(1,0) (5,0)	(0,-10)
3	g(x) = x^2 + 2x -8	Dal	(-1,-9)	(2,0) (-4,0)	(0,-8)
4	k(x) = $(x+1)^2$ – 9	Dal	(-1,-9)	(2,0) (-4,0)	(0,-8)
5	m(x) =-2 $(x-3)^2$ + 8	Berg	(3,8)	(1,0) (5,0)	(0,-10)
6	n(x) = ½ x^2 – x – 7½	Dal	(1,-8)	(5,0) (-3,0)	(0,-7 ½)
7	o(x) = -1(x-2)·(x+6)	Berg	(-2,16)	(2,0) (-6,0)	(0,12)
8	p(x) = -2x^2 +12x -10	Berg	(3,8)	(1,0) (5,0)	(0,-10)
9	q(x) = -x^2 - 4x + 12	Berg	(-2,16)	(2,0) (-6,0)	(0,12)
10	r(x) = ½ (x-5)(x+3)	Dal	(1,-8)	(5,0) (-3,0)	(0,-7 ½)
11	s(x) = ½ $(x-1)^2$ – 8	Dal	(1,-8)	(5,0) (-3,0)	(0,-7 ½)
12	t(x) = (x-2)(x+4)	Dal	(-1,-9)	(2,0) (-4,0)	(0,-8)

Gebruik in de klas

Voorkennis leerlingen:
Leerlingen moeten kunnen ontbinden in factoren en kwadraat-afsplitsen.

Voorbereiding docent:
De docent print de kaartjes voor ieder groepje en legt deze klaar op de tafels.
Op het bord komt de vraag: ‘wat valt op’.

Hoe uit te voeren?:

- Leerlingen krijgen allemaal een blad (zie bijlage) waarop ze de uitwerkingen kunnen invoeren.
  Ook op de kaartjes zelf kunnen de leerlingen de eigenschappen noteren.
- Leerlingen krijgen de opdracht om de eigenschappen op te schrijven (op de kaartjes zelf en/of op het overzichtsblad; leerlingen hierin zelf laten kiezen)
- Na ongeveer 10 minuten laat je de leerlingen benoemen wat er zoal opvalt.
  Door met de kaartjes te schuiven kunnen de leerlingen de bijzondere vormen bij elkaar leggen en overeenkomsten sneller zien.
- De uitkomst zou moeten zijn dat Leerlingen leren te doorzien dat bij verschillende verschijningsvormen verschillende dingen makkelijk zijn af te lezen.
  Daarnaast zijn de voorbeelden zo gekozen dat er 4 keer eenzelfde grafiek wordt gevraagd met een andere manier van noteren. Hierdoor gaat het snel en zien leerlingen ook dat een andere manier van opschrijven niet per definitie een andere grafiek hoeft voor te stellen.

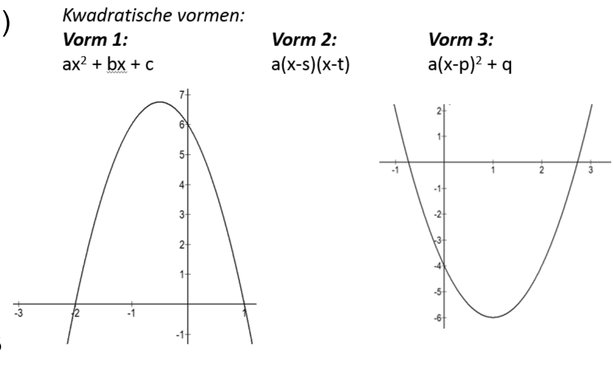
  Zaken die in de uitwerking van de opdracht zitten zijn:
  f(x) a(x-s)(x-t) ontbinden
  a: bepaalt berg/ dal
  s en t: snijpunten met x-as (s,0) en (t,0)

  f(x) = a$(x-p)^2$ + q kwadraat afsplitsen
  a: bepaalt berg/dal
  p en q: top (p,q)

  f(x) = ax^2 + bx + c algemene vorm
  a: bepaalt berg/dal
  c: Snijpunt met y-as (0,c)

- Bij het bespreken van de uitwerkingen kun je gebruik maken van het bijgeleverde excel-bestand.
  Hierin staan eerst de opdracht, vervolgens stapsgewijs de uitwerking alsook een dia (dia nr 4) waarin leerlingen zelf de bijzonderheden kunnen noemen.

Wat hierna?:
Nadat deze opdracht klaar is (na ongeveer 20 minuten) kunnen leerlingen de ‘finale’ opdracht krijgen.
Deze is te vinden in zowel de bijlage als op dia 5 van het bijgevoegde excel-bestand.
De opdracht hierbij is vrij simpel:
Kies een geschikte vorm en maak het functievoorschrift bij de gegeven grafieken.



De uitwerking zou moeten zijn:
Snijpunten x-as zijn (-2,0) en (1,0) dus a(x+2)(x-1)
Grafiek gaat door (0,6) dus a(0+2)(0-1) = 6 → a = -3
Top bij (1,-6) Dus $g(x) = a(x-1)^2 – 6$
Grafiek gaat door (0,-4) dus $a(0-1)^2 – 6 = -4$ → a = 2 $g(x) = 2(x-1)^2 - 6$

Mogelijkheden tot differentiatie:
Leerlingen die snel gaan, kun je ook een extra lastige opdracht geven (zie dia 6 bij het excel-bestand).
Ze zullen tot de conclusie komen dat eigenlijk geen van de 3 kwadratische vormen direct een oplossing biedt. Hierbij moeten leerlingen uiteindelijk een stelsel vergelijkingen oplossen.
De uitwerking hierbij is:
de andere vormen zijn niet handig, aangezien je geen top of snijpunten met x-as af kunt lezen.
Snijpunt met de y-as is (0,4) dus c= 4
Twee punten aflezen: (1,22) en (8,36)
Invullen levert:
$22 = a·1^2 + b·1 + 4$ → a + b = 22 → b = 18 – a
$36 = a·8^2 + b·8 + 4$ → 64a + 8b = 32 → b = -8a + 4
Samenvoegen levert: 18 -a = -8a + 4 → a = -2 en dus b = 20
Dus $y = -2x^2 + 20x + 4$

Je kunt leerlingen ook zelf een grafiek laten bedenken en daarbij een functievoorschrift laten zoeken.

Tips:
- In verband met het in de gaten houden van de tijd kun je er voor kiezen om de helft van de groepjes te laten beginnen met de linker grafiek en de andere helft met de rechter grafiek.
  Zo zijn ze allemaal aan de slag en kun je een stuk tijd besparen door halverwege de les stil te leggen en te bespreken.
- Je kunt bij het bespreken verschillende groepjes verschillende vormen laten beantwoorden, zodat iedereen iets anders bekijkt.
- De eerste opdracht duurt ongeveer 20 minuten, de tweede ongeveer een kwartier.

Vragen en hints om leerlingen te helpen:
- Zie je overeenkomsten?
- Welke eigenschappen komen terug in het functievoorschrift
- Kloppen de eigenschappen ook bij de andere voorbeelden?
- Zou je een soort algemene vorm kunnen bedenken?
- Wanneer kun je welke vorm nu eigenlijk het beste gebruiken?

- Bij welke vorm heb je eigenlijk de minste gegevens nodig?
  Hoeveel gegevens heb je hoe dan ook nodig om een functievoorschrift te kunnen maken?
  Welke vorm is het handigst? En waarom?

Appendix G.2. Questions for and answers by the teacher who tested this MTA

This Appendix contains questions to Dirk Boleij, the teacher who performed this MTA lesson, and his answers. I thank him for permission to reprint these.

The discussion is in Dutch.

It appears that Boleij first presented his students with his own classification scheme, that he calls "WOKA-NOVA". Thus we should be careful in comparing results over different implementations.

Inleiding tot de vragen en antwoorden

De vragen over de WDA t.a.v. kwadratische functies geven een complexer beeld dan verwacht. Dirk Boleij, de uitvoerder bij het NRO-project, heeft tevens een eigen aanpak ontwikkeld in afwijking van of in aanvulling op het boek.

Deze aanpak wordt "WOKA-NOVA" genoemd naar de afkortingen daarin. In deze aanpak krijgen leerlingen een enkele pagina met een overzicht van de verschillende kwadratische vormen en hoe te handelen bij veelvoorkomende vragen. Deze aanpak ziet er erg goed uit, en hopelijk wil Boleij dit ooit publiceren.

Klaarblijkelijk krijgen de leerlingen eerst deze uitleg, voordat de WDA wordt gedaan. Boleij schrijft: "*Deze leerlingen hebben deze uitleg dus niet her-ontdekt, ze hebben de kwadratische vormen nog niet eerder zo gezien!*"

Date: Thu, 02 Jun 2016 TC -> DB

Geachte collega Boleij,

Ik zie dat u meedeed aan dit NRO-project,

https://www.nro.nl/kb/405-14-502-wiskundige-denkactiviteit-in-wiskunde-op-havo-en-vwo

http://www.fisme.science.uu.nl/toepassingen/28403

en dan t.a.v. de kwadratische functie, waarover ik dit artikeltje schreef in de WiskundE-brief:

http://www.wiskundebrief.nl/738.htm#5

Ten eerste mijn compliment dat dit een heel aardige les is. Mijn vragen voor u zijn:

(1) Bevat mijn bericht in de WiskundE-brief nu nog iets nieuws voor u, en zou u kunnen aanduiden wat dan voor u nog nieuw is ?

(2) Heeft u meer gedaan dan ontwikkelen van deze WDA, zoals nulmeting en toets achteraf, met vergelijken van welke methode nu productiever is ? Of staat dit nog in de kinderschoenen ?

(3) Mijn vermoeden is dat het wiskundig productiever is om eerst klassikaal te werken volgens mijn voorgestelde didactiek uit de WiskundE-brief, zodat ook zwakke leerlingen de kernbegrippen kunnen vatten, en dat WDA gericht wordt op andere zaken, zoals voor zwakke leerlingen het herontdekken van wat ze reeds klassikaal hebben gekregen, waarvan uw filmpje een mooi voorbeeld is, en voor sterkere leerlingen toch wat anders, zoals de afleidingen tussen de verschillende vormen en de abc-formule (het bewijzen). Mee eens ? Voor die afleidingen, zie de links bij het artikeltje in de WiskundE-brief.

PM. Overigens vind ik "denkactiviteit" een verkeerde term, want er is verschil tussen doen en denken. Wat is er mis met gewoon "denken". U zegt ook terecht dat "wiskundig denken" alleen maar denken is (over wiskundige vragen).

Met vriendelijke groet,

Thomas Cool / Thomas Colignatus
Econometrist en leraar wiskunde
Scheveningen

Date: Thu, 2 Jun 2016 DB -> TC

From: "Dirk Boleij"
To: Thomas Cool / Thomas Colignatus
Subject: RE: Aan wiskundeleraar Dirk Boleij n.a.v. NRO-project

Beste heer Cool,

Voor een ' antwoord' op uw vragen, eerst een korte 'schets':
Ik heb onlangs deze ' les' in ath3 ' weer' gegeven.
Ik link de vorm van de grafieken hierbij aan een theorie die ik in een eerder stadium met de leerlingen bespreek bij het oplossen van vergelijkingen, dat sluit eigenlijk mooi aan.
Helaas sluit het niet aan bij het door u geschetste verhaal: denken ipv trucs :)
Ik leer bij het oplossen van vergelijkingen de leerlingen namelijk de 'truc' WOKA-NOVA.
Klinkt misschien vaag, maar in het kort komt het neer op:

Slechts één 'x' --> Weegschaalmethode (links en rechts hetzelfde doen tot je klaar bent)
Meerdere ' x'-en --> Probeer te Ontbinden , lukt dit niet, dan Kwadraat afsplitsen en als echt niets meer lukt de Abc formules. W-->O-->K-->A.
De NOVA staat voor de stappen bij O.K.A..
Eerst Nul maken, Ontbinden, Vereenvoudigen Antwoord geven .
in schema-vorm:

```
   N
W OK A
   V
   A
```

Leerlingen slikken dit als zoete koek (maar het blijft een ezelsbruggetje).

Ik benadruk dan ook dat de ABC - formule eigenlijk niet veel 'toevoegt'.

Nu denkt u wellicht: wat heeft dit te maken met het voorbeeld?

Maar voor leerlingen is vervolgens de vorm van de kwadratische functie heel makkelijk terug te zien in de verschillende letters

a(x-b)(x-c)	Ontbinden
a(x-b)^2 + c	Kwadraat afsplitsen
ax^2 + bx + c	Abc formule

Daarbij zien ze ook dat de Abc-formule pas een laatste ' redmiddel' lijkt, en (toeval of niet) de laatste vorm is eigenlijk de vorm waarin het 'minst' af te lezen is. Kun je de top aflezen, dan weet je direct 2 van de 3 variabelen, terwijl het snijpunt met de y-as in de laatste vorm slechts de ' c' oplevert.

Vanuit deze 'koppeling' is het 'waarom' een bepaalde methode te gebruiken ook logischer voor leerlingen.

Ik gebruik overigens niet de term ' draaipunt'. Maar wellicht is het een logisch woord.

(opvallend is dat kwadratische vormen binnen Moderne Wiskunde pas sinds de nieuwe ' editie' een duidelijke plaats heeft gekregen. Ik merk ook dat leerlingen deze vormen snel onder de knie hebben (en dat de les die ik geef aan de hand van het blad ontzettend snel verloopt, en de leerlingen het ook vrijwel direct heel snel begrijpen!!)

Aangezien ik dit echt een leuk onderdeel in klas 3 vind, kan ik hier nog wel verder over doordraven, maar ik zal antwoord trachten te geven op uw vragen:

(1) Bevat mijn bericht in de WiskundE-brief nu nog iets nieuws voor u, en zou u kunnen aanduiden wat dan voor u nog nieuw is ?

De term draaipunt is voor mij nieuw. Ik vraag me af of u deze verschuivingen in de klas (3e??) ook specifiek benoemd; of dit pas in een hoger jaar doet. In de opbouw bij de methode Moderne Wiskunde worden de transformaties vaak een stukje doorgeschoven richting klas 4. Ik behandel de kwadratische vormen dan ook vooral met het idee: hoe kan ik snel een formule bij een grafiek maken, of : hoe lees ik snel bijzondere punten van een functie bij een grafiek af.

Nieuw zou dan zijn: het directer koppelen van de 'transformaties' aan de verschillende vormen.

(2) Heeft u meer gedaan dan ontwikkelen van deze WDA, zoals nulmeting en toets achteraf, met vergelijken van welke methode nu productiever is ? Of staat dit nog in de kinderschoenen ?

Het was een kortlopend onderzoek, waarbij we wel een voor- en natoets hebben gehad, maar niet specifiek op deze opdracht.;Te kleine testgroep, te weinig tijd enz.
Dus in dat opzicht staat dit nog in de kinderschoenen.
Wel merk ik dat (toevallig met dit hoofdstuk, waarbij het viel in een periode met veel uitval en weinig tijd) zo'n heel hoofdstuk direct met deze opdracht door leerlingen (alle) begrepen wordt, en dus prima in 1-2 lessen behandeld kan worden. Je hebt het boek dus absoluut niet altijd nodig om een stuk theorie helder te krijgen.
Ik ben er van overtuigd dat dit ook met andere onderwerpen kan.

(3) Mijn vermoeden is dat het wiskundig productiever is om eerst klassikaal te werken volgens mijn voorgestelde didactiek uit de WiskundE-brief, zodat ook zwakke leerlingen de kernbegrippen kunnen vatten, en dat WDA gericht wordt op andere zaken, zoals voor zwakke leerlingen het herontdekken van wat ze reeds klassikaal hebben gekregen, waarvan uw

filmpje een mooi voorbeeld is, en voor sterkere leerlingen toch wat anders, zoals de afleidingen tussen de verschillende vormen en de abc-formule (het bewijzen). Mee eens ?

Ik denk dat het hierbij heel erg afhankelijk is van het onderwerp, de opdracht en zelfs: wat wordt verstaan onder WDA. Ik merk dat sommige mensen een 'verhaaltjes-som' al zien als wiskundig-denk-actief. Het denkactieve element is echter sterk afhankelijk van wat leerlingen al weten, de voorkennis dus.
Als bijvoorbeeld een kwadratische vorm voor leerlingen echt iets nieuws is, begrijp ik dat het voor de zwakkere leerlingen lastig kan zijn om zomaar iets te ontdekken.
Maar ik denk dat bijvoorbeeld deze opdracht ook voor de hele zwakke leerlingen vrij toegankelijk is.
Deze leerlingen hebben deze uitleg dus niet her-ontdekt, ze hebben de kwadratische vormen nog niet eerder zo gezien!
De afleidingen van de vormen vind ik dan inderdaad wel erg denkactief voor de betere leerling: waarom zou je eerst snijpunten met de x-as bepalen (abc formule), vervolgens daar tussenin gaan zitten en tenslotte dat getal in gaan vullen, als je heel ' elegant' de formule om kunt bouwen naar een vorm waarin je de top direct kunt aflezen? Ze ondervinden daadwerkelijk dat ze een stap zetten die netjes werkt en ook nog eens effectief is.
(Ik leer de leerlingen niet aan dat de top -b/2a is, in ieder geval zeker niet klassikaal, dat vind ik namelijk een truc die ze te vaak toepassen zonder echt ' inzicht' in de materie te hebben.

Voor die afleidingen, zie de links bij het artikeltje in de WiskundE-brief.
Ik vind het wel interessant om te zien dat u (toch??) start met de vorm $a(x-b)^2+c$ (andere letters uiteraard) en van daaruit de ABC formule herleidt.
Terwijl (als de leerlingen hier al om vragen) ik de ABC formule vaak laat zien doormiddel van kwadraatafsplitsen zonder 'grafieken'
$ax^2+bx+c = 0$
$x^2 + b/ax + c/a = 0$
$(x + b/2a)^2$ *enz enz* *(en dan zie je dus ook weer het belang van kwadraatafsplitsen boven de abc-formule)*
U gebruikt dus daadwerkelijk de link met formules en transformaties, maar uit beide manieren blijkt: de abc formule is een truc die je eigenlijk zonder kwadraatafsplitsen (of die 'vorm') niet kunt laten zien, dus eigenlijk is die abc-formule ook niet echt ' interessant' .

Goed om eens andere herleidingen te zien!!

PM. Overigens vind ik "denkactiviteit" een verkeerde term, want er is verschil tussen doen en denken. Wat is er mis met gewoon "denken". U zegt ook terecht dat "wiskundig denken" alleen maar denken is (over wiskundige vragen).
Ik heb de term niet bedacht (gelukkig!). Ik heb af en toe ook het idee dat de term denkactiviteit te pas en te onpas wordt gebruikt om aan te tonen dat de methodes ' met de huidige tijd' meegaan.
Maar als ik dan soms opdrachten zie die dan 'denkactief' zouden moeten zijn, denk vaak: dit is gewoon een verhaaltjessom waarbij leerlingen, die gewoon de basisvaardigheden beheersen, makkelijk uitkomen zonder veel te hoeven denken.
Het is juist goed om van de bebaande paden af te gaan, maar uiteraard vergt dat van een docent wel veel (creativi-)teit.

Met vriendelijke groet,

Thomas Cool / Thomas Colignatus
Econometrist en leraar wiskunde
Scheveningen

Met vriendelijke groet
Dirk Boleij

p.s. Ik hoop dat mijn 'antwoorden' een beetje aan uw verwachting voldoen. Ik heb nog niet eerder op deze manier vragen gekregen over 'gepubliceerde' producten, wel interessant!

Ook leuk dat het materiaal daadwerkelijk bekeken wordt!

Date: Thu, 02 Jun 2016 TC -> DB

To: "Dirk Boleij"
From: Thomas Cool / Thomas Colignatus
Subject: RE: Aan wiskundeleraar Dirk Boleij n.a.v. NRO-project

Geachte heer Boleij,

Hartelijk dank voor uw snelle antwoord. Ik wil het even laten bezinken, wat enige tijd kan duren. Bijv. kende ik WOKA-NOVA niet, en ik weet nog niet wat daarvan te denken ....

Voor het goede begrip: ik sta niet meer voor de klas, maar wil de opgedane ervaring wel gebruiken voor het zoeken naar verbeteringen.

Inderdaad is het denken afhankelijk van de voorkennis. Om goed denken te stimuleren, is het sorteren van de voorkennis dan van belang.

Ik zocht de basisvorm in x^2, en vervolgens het verplaatsen. Dan moet verplaatsen tot de voorkennis behoren.

Het valt me op dat u ontbinden als eerste noemt. Misschien is het te combineren. Bijv. bij x^2 zijn er twee x-en, schrijf die als x x', en die zou je ook apart kunnen verplaatsen, zodat x x' -> (x +2) (x + 5), en zo heb je het ontbinden in factoren.

Het doet me deugd te horen dat leerlingen het snel oppakken. Het naast elkaar zetten van verschillende vormen en daarover nadenken, is belangrijk, en het is wonderlijk dat dit niet traditioneel al gedaan werd.

Met dank, en ik reageer later dieper,

Thomas Cool / Thomas Colignatus

Date: Mon, 31 Oct 2016 TC -> DB

To: "Dirk Boleij"
From: Thomas Cool / Thomas Colignatus
Subject: N.a.v. WDA kwadratische functies

Geachte collega Boleij,

Op de komende NVvW jaarvergadering / studiedag hoop ik een workshop C6 te verzorgen. In het kader daarvan heb ik inmiddels wat meer nagedacht over de WDA.

(1) Mijn voorlopige evaluatie staat hier:

http://thomascool.eu/Papers/Math/2016-10-31-MTA.pdf

(2) Ik heb nog eens gekeken naar mijn vragen en uw antwoorden t.a.v. uw WDA t.a.v. kwadratische functies.

Eigenlijk zouden die heel goed kunnen passen bij bovenstaande evaluatie, indien geredigeerd op publicatie.

Zou u bereid zijn om te overwegen of u uw tekst zo zou kunnen aanpassen, dat u publicatie daarvan toestaat, en dat ik dan mijn vragen en uw aangepaste antwoorden kan opnemen op mijn website voor de documentatie van mijn evaluatie van WDA ?

Eventueel is uw reactie dat u hiervoor geen tijd heeft, en ook niet goed kan overzien wat wel en niet relevant is voor mijn vraagstellingen. In dat geval zou ik een concept kunnen maken, en het nodige redigeren.

U heeft dan nog de optie om met name genoemd te worden, of dat ik een geanonimiseerde versie maak, die ook niet meer naar u is terug te leiden, zodat u dan een van de naamloze docenten wordt die deze WDA ook gebruikt hebben. Ook zo'n geanonimiseerd concept zou ik aan u voorleggen, zodat u kunt nagaan dat het inderdaad niet naar u terugverwijst.

Misschien wilt u mij zeggen hoe u hierover denkt.

(3) Overigens geeft een google mij geen link voor WOKA-NOVA. Heeft u er een ?

(4) Mijn voorlopige evaluatie onder (1) t.a.v. WDA is kritisch, en ik hoop niet dat het u afschrikt t.a.v. (2). Op zich zou ik ook een reactie op (1) natuurlijk waarderen, maar mijn hoop is dat sowieso (2) afgesproken kan worden, liefst met voorrang zodat het er netjes bijstaat.

Nogmaals met dank,

Thomas Cool / Thomas Colignatus
Econometrist en leraar wiskunde
Scheveningen

Date: Fri, 4 Nov 2016 DB -> TC

From: "Dirk Boleij"
To: "Thomas Cool / Thomas Colignatus"
Subject: Re: N.a.v. WDA kwadratische functies

Geachte heer Cool,

Uiteraard mag u mijn opmerkingen best gebruiken in uw publicatie.
Ik weet echter niet precies wat uw vraag op dit moment is.

Ik heb zelf nauwelijks ervaring met publicaties e.d. en dus ook niet met het wel of niet benoemen van iemands naam. Persoonlijk heb ik geen moeite met zowel het wel als niet vernoemen van mijn naam.

Mijn opmerkingen mbt WDA zijn gebaseerd op de ervaringen die ik heb opgedaan bij het (samen met P. Drijvers, ik zie dat u zijn publicaties ook heeft toegevoegd) door een 7 tal docenten uitgevoerde praktijkonderzoek. Overigens lag de nadruk hierbij meer op uitvoeren en ontwikkelen dan op echte data-analyse.

Dat WOKA-NOVA niet op internet te vinden in begrijp ik, aangezien ik het zelf bedacht heb (en niet gepubliceerd) het is gewoon een verzonnen ezelsbruggetje, wat wel in de praktijk erg fijn blijkt te werken.

In de bijlage heb ik een samenvatting toegeveogd, zoals ik ze aan mijn leerlingen geef.

Graag hoor ik van u in hoeverre ik u nog kan helpen.

Ik merk dat het werkveld waarin u zich begeeft (volgens mij meer onderzoek/ publicaties/ theorieen) en waarin ik zit (vaak dingen op een eenvoudige manier aan leerlingen uitleggen, maar ze wel uitdagen tot nadenken) er op papier toch wel iets anders uitziet.

Met vriendelijke groet,

Dirk Boleij

Date: Mon, 14 Nov 2016 DB -> TC

From: "Dirk Boleij"
To: "Thomas Cool / Thomas Colignatus"
Subject: Re: N.a.v. WDA kwadratische functies

Ik ben een tijdje weg geweest, vandaar pas deze late reactie.

In de door u gestuurde bijlagen zie ik geen gekke dingen (het zijn gewoon de geredigeerde mails die wij elkaar stuurden..) de tekst is volgens mij kloppend.

Ik heb inderdaad eerst WOKA-NOVA uitgelegd in de klassen en vervolgens zijn we pas begonnen met de WDA opdrachten.

Ik neem aan dat u deze 'mailwisseling' meer als bijlage gebruikt dan daadwerkelijk in een publicatie laat drukken? Ik bedoel: de daadwerkelijke mailwisseling op zich lijkt mij niet heel interessant voor mensen die iets willen weten over WDA (en of WOKA-NOVA) ?

Ik heb eerlijk gezegd niet nagedacht over publicatie van het 'trucje' WOKA-NOVA.
Ik ben al blij wanneer het de leerlingen helpt om op een gestructureerde manier vergelijkingen op te lossen. Zoals al eerder vermeld: het publiceren is voor mij een 'ver van mijn bed show' .

Met vriendelijke groet,

Dirk Boleij

Appendix: Boleij's WOKA-NOVA classification, slightly adapted

<table>
<tr><th colspan="4">THEORIE</th></tr>
<tr><td>• Lineaire vergelijkingen
• Vergelijkingen met één variabele</td><td colspan="3">• Kwadratische vergelijkingen</td></tr>
<tr><td></td><td>Nul maken</td><td></td><td></td></tr>
<tr><td>Weegschaalmethode (steeds omgekeerde handeling uitvoeren om iets weg te werken)</td><td>Ontbinden</td><td>Kwadraat afsplitsen</td><td>ABC formule</td></tr>
<tr><td></td><td>Vereenvoudigen</td><td></td><td></td></tr>
<tr><td></td><td>Antwoorden</td><td></td><td></td></tr>
<tr><th colspan="4">VOORBEELDEN</th></tr>
<tr><td>Lineair:
$3(x - 2) = 5 - (2x - 6)$
$3x - 6 = 5 - 2x + 6$
$5x = 17$
$x = 17/5 = 3\ 2/5$

Kwadratisch:
$(2x-5)^2 = 16$
$2x - 5 = 4$ of $2x - 5 = -4$
$2x = 9$ of $2x = 1$
$x = 4\frac{1}{2}$ of $x = \frac{1}{2}$

Wortel:
$2\sqrt{x-3} + 3 = 11$
$2\sqrt{x-3} = 8$
$\sqrt{x-3} = 4$
$x - 3 = 16$
$x = 19$ (voldoet)</td><td>Tweeterm:
$2x^2 = 6x$
$2x^2 - 6x = 0$ N
$2x(x-3) = 0$ O
$2x = 0$ of $x - 3 = 0$ V
$x = 0$ of $x = 3$ A

Drieterm:
$x^2 + 3x = 10$
$x^2 + 3x - 10 = 0$ N
$(x+5)(x-2) = 0$ O
$x+5 = 0$ of $x - 2 = 0$ V
$x = -5$ of $x = 2$ A</td><td>$x^2 + 6x - 3 = 0$
$(x+3)^2 - 12 = 0$
$(x+3)^2 = 12$
$x+3 = \sqrt{12}$ of $x+3 = -\sqrt{12}$
$x + 3 = 2\sqrt{3}$ of $x+3 = -2\sqrt{3}$
$x = -3+2\sqrt{3}$ of $x = -3-2\sqrt{3}$</td><td>$2x^2 - 3x - 4 = 0$

$D = (-3)^2 - 4 \cdot 2 \cdot -4 = 41$

$x = \frac{3+\sqrt{41}}{4}$
of $x = \frac{3+\sqrt{41}}{4}$

$x = \frac{3}{4} + \frac{1}{4}\sqrt{41}$
of $x = \frac{3}{4} - \frac{1}{4}\sqrt{41}$</td></tr>
<tr><th colspan="4">THEORIE</th></tr>
<tr><td colspan="2">VORM:
$A^2 = B^2$
oplossingen:
A = B of A = -B</td><td colspan="2">VORM:
A · B = A · C
oplossingen:
B = C of A = 0</td></tr>
<tr><th colspan="4">VOORBEELDEN</th></tr>
<tr><td colspan="2">Eenvoudig:
$x^2 = 9$ (eigenlijk $x^2 = 3^2$)
$x = 3$ of $x = -3$

Moeilijker:
$(2x - 4)^2 = (4x - 6)^2$
$2x - 4 = 4x - 6$ of $2x - 4 = -4x + 6$
$2x = 2$ of $6x = 10$
$x = 1$ of $x = 10/6 = 1\ 2/3$</td><td colspan="2">Eenvoudig:
$x \cdot (2x+5) = x \cdot (x - 6)$
$2x + 5 = x - 6$ of $x = 0$
$x = -11$ of $x = 0$

Moeilijker:
$(2x-4)^2 = (2x-4) \cdot (-4x+3)$
$2x - 4 = -4x + 3$ of $2x - 4 = 0$
$6x = 7$ of $2x = 4$
$x = 7/6 = 1\ 1/6$ of $x = 2$</td></tr>
</table>

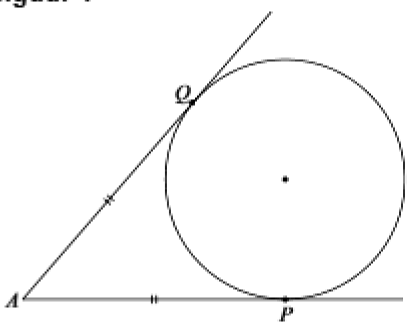
Appendix H. Pilot VWO B 2014-1, question 3

Question [151]

Cirkels in een driehoek

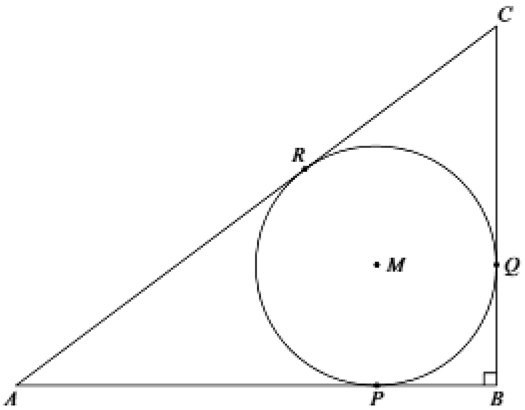
Als vanuit een punt A buiten een cirkel de twee raaklijnen aan die cirkel getrokken worden, dan zijn de afstanden van A tot de twee raakpunten P en Q even groot. In figuur 1 geldt dus $AP = AQ$.
Deze eigenschap mag je in deze opgave gebruiken.

figuur 1



Gegeven is een rechthoekige driehoek ABC met rechthoekszijden $AB = 4$ en $BC = 3$. De ingeschreven cirkel van driehoek ABC raakt de zijden van de driehoek in P, Q en R. M is het middelpunt van deze cirkel.
Zie figuur 2.

figuur 2



De straal van de ingeschreven cirkel van driehoek ABC is 1.

4p 3 Bewijs dit.

[151] http://static.examenblad.nl/9336114/d/ex2014/vw-1025-f-14-1-o.pdf

Grading [152]

3 **maximumscore 4**

- (Uit de stelling van Pythagoras of met 3-4-5 driehoek volgt) $AC = 5$ — 1
- Noem de straal van de cirkel x, dan $BP = BQ = x$ — 1
- $AR = AP = 4 - x$ en $CR = CQ = 3 - x$ — 1
- ($AC = AR + CR$, dus) $(4-x)+(3-x)=5$ geeft $x = 1$ — 1

of

- (Uit de stelling van Pythagoras of met 3-4-5 driehoek volgt) $AC = 5$ — 1
- oppervlakte(ΔABC) = oppervlakte(ΔABM) + oppervlakte(ΔBCM) + oppervlakte(ΔCAM) — 1
- Dit geeft $\frac{1}{2} \cdot AB \cdot BC = \frac{1}{2} \cdot AB \cdot x + \frac{1}{2} \cdot BC \cdot x + \frac{1}{2} \cdot CA \cdot x$ — 1
- $\frac{1}{2} \cdot 4 \cdot 3 = \frac{1}{2} \cdot 4 \cdot x + \frac{1}{2} \cdot 3 \cdot x + \frac{1}{2} \cdot 5 \cdot x$ geeft $x = 1$ — 1

Discussion by Buitenhuis (2016:83) [153]

Vraag 3: Modelleren als proces (Mp) en probleemoplossen (P).
Het invoeren van de variabele *x*, waardoor het meetkundige probleem een algebra probleem wordt is modelleren (Mp).
Er wordt een stelling gegeven, die gebruikt mag worden. Dit stuurt de leerling richting een bepaalde oplossingsweg. Door verschillende gegevens te combineren creëert de leerling een oplossingsweg (P), welke van te voren niet meteen zichtbaar was.
Er zijn twee wezenlijk verschillende oplossingswegen (P): de eerste manier maakt gebruik van lengtes en de tweede manier maakt gebruik van het berekenen van de oppervlakte.

Correctievoorschrift
P: oplossingsweg 1 / oplossingsweg 2
oplossingsweg 1 Mp: Het stellen van variabele *x* (onderdeel van bolletje 2)
oplossingsweg 1 P: alle vier de bolletjes vormen samen de oplossingsstrategie
oplossingsweg 2 Mp: Het stellen van variabele *x* (onderdeel van bolletje 3)
oplossingsweg 2 P: alle vier de bolletjes vormen samen de oplossingsstrategie

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152 http://static.examenblad.nl/9336114/d/ex2014/vw-1025-f-14-1-c.pdf
153 http://dspace.library.uu.nl/bitstream/handle/1874/318236/Onderzoek%20naar%20WDA%20in%20pilotexamens_Hanneke%20Kodde_juli%202015.pdf?sequence=2

Appendix I. Attention for didactics blocked by focus on MTA

Two persons asked to "organise" the NVvW annual study day decide amongst themselves not to offer teachers the choice to attend a workshop on innovations in didactics.

They think that teachers would be interested mainly in the new exam programme.

Why not simply offer the workshops as options, so that teachers can decide for themselves ?

Date: Sun, 2 Aug 2015

From: [Organisers]
To: "Thomas Cool / Thomas Colignatus"
Subject: RE: Twee voorstellen voor workshops NVvW Studiedag 2015

Beste Thomas,

dank voor je enthouisiaste reactie op de jaarvergadering. Het afgelopen jaar is je eerste voorstel niet in de prijzen gevallen. Blijkbaar hebben de aanwezige wiskundedocenten andere concerns. Op dit moment worden we overladen met workshops. Jouw tweede voorstel bevat een mooi onderwerp, dat misschien wel een plek zou moeten krijgen in het wiskundecurriculum van het VO. Het nieuwe curriculum van 2015 bevat dit onderwerp echter niet. De workshops die op dit moment voor de wiskundedocent van belang zijn, gaan over het het nieuwe curriculum. Vandaar dat we je voorstel niet kunnen meenemen voor de komende jaarvergadering.

Met vriendelijke groet, [Organisers]

Date: Wed, 29 Jul 2015

To: [Organisers]
From: Thomas Cool / Thomas Colignatus
Subject: Twee voorstellen voor workshops NVvW Studiedag 2015
Cc: l.j.b.wesker-elzinga at hva.nl

Beste [Organisers],

Bijgaand twee voorstellen voor workshops:

(1) Herhaling van vorig jaar: Strakke goniometie op omtrek 1 met Xur[α] = Cos[Θ α] en Yur[α] = Sin[Θ α]

(2) Verzamelingenleer, getaltheorie en oneindigheid

Hartelijke groet,

Thomas

(Ad 1) Strakke goniometie op omtrek 1 met xur[α] = cos[Θ α] en yur[α] = sin[Θ α]

door Thomas Cool / Thomas Colignatus

Goniometrie kan strakker. Voor SOSCASTOA moet je zoeken en draaien: dus begin met de eenheidscirkel met straal 1 met vaste oriëntatie. Sin[φ] = OS is een op te lossen vergelijking en geen functiedeclaratie. (Vergelijk f[x] = b x + c.) De werkstructuur φ = ArcTan[y/x] toont dat je van coördinaten naar de boog gaat. Ook zo'n wonder: de 360 graden en Archi = Θ = 2π radialen vallen uit de lucht. Helder is de maat van het platte vlak zelf. Dit geeft de cirkel met straal 1 / Θ en omtrek 1, met het aantal draaien α. De coördinaten op de cirkel met straal 1 zijn dan {x, y} = {xur[α], yur[α]} = {cos[Θ α], sin[Θ α]}. Zie http://thomascool.eu/Papers/COTP/Index.html

[Was included in 2014: session E2: https://www.nvvw.nl/18045/thema-e-diversen but there was attention only by 1 person. [154]]

(Ad 2) Verzamelingenleer, getaltheorie en oneindigheid

door Thomas Cool / Thomas Colignatus

Verzamelingenleer en getaltheorie ontbreken in het programma en eindexamen vwo-b maar zouden daar thuishoren. Een verklaring voor dat ontbreken is de verkeerde aanpak in 1960-1975 van de "New Math". Een aanvullende verklaring is dat verzamelingenleer snel tot Cantor's theorema over de machtverzameling leidt, met afleidingen omtrent oneindigheid en transfinieten: wat dan snel moeilijk en weinig practisch is. Echter, de ZFC axioma's blijken inconsistent en Cantor's theorema weerlegd, en aldus is een verzamelingenleer zonder transfinieten mogelijk. Zie http://thomascool.eu/Papers/FMNAI/Index.html

[154] http://thomascool.eu/Papers/AardigeGetallen/2016-04-12-stalker-kat-op-webmaster-spek.pdf

Appendix J. Samenvattende voorlopige evaluatie: Problemen en vragen t.a.v. de "wiskundige denkactiviteiten" (WDA) in de examens vanaf 2017 (HAVO) en 2018 (VWO), volgend op acht jaar cTWO 2004-2012"

Wat ik erover lees komt bij mij over als een wanproduct, waarbij niet-wetenschappers (wiskundigen) de empirische cyclus in de wetenschap willen nabouwen. Een wanproduct, waarbij *zowel* het klassieke wiskundige bewijzen *als* de "toegepaste wiskunde" *als* de overstap naar werkelijk empirisch onderzoek niet tot hun recht komen.

De WDA zijn slecht gedefinieerd, geen *evidence based education,* en het hoofdrapport is afgesloten en aan de minister aangeboden terwijl er nu nog aan de implementatie wordt gewerkt om te bepalen wat er nu precies besloten is.

Op het eindexamen moet je toetsen op zaken waarop leerlingen zich redelijkerwijs kunnen voorbereiden. Een toets op inzicht heeft daar geen plaats. Bevorderen van inzicht heeft een plaats in de voorbereiding op het eindexamen, omdat je dan alle vragen makkelijker aankunt, maar inzicht zelf toetsen is een niet aan de orde.

Een kandidaat voor het eindexamen hoeft een vraag met WDA niet te beantwoorden om toch te slagen, als de rest maar goed is, zodat WDA dient voor het cijfer vanaf wellicht 7.5. Maar krijgt de kandidaat dan wel een correcte kans op slagen ? Moeten niet alle vragen niet-WDA zijn, om de overgrote meerderheid een redelijke kans te bieden om daarop minstens een 5.5 te scoren ? Is WDA echt essentieel om daarna een universitaire studie te doen ? Lag de bottleneck niet eerder bij algebraische vaardigheden en dergelijke ?

Ik vrees dat de ***term*** WDA is ingevoerd *mede* door de ideologen van het "realistische wiskunde-onderwijs". Waar voorheen "contexten" werden gebruikt en deze kritiek kregen als verwarrend voor het zuiver wiskundig denken, wordt gedaan alsof er iets nieuws is waardoor toch weer contexten kunnen worden gebruikt maar nu onder een andere naam.

Het effect zou wellicht het grootst kunnen zijn voor het primair onderwijs (PO), wanneer gesteld wordt dat de contexten aldaar toch ook weer WDA op dat niveau zouden zijn.

WDA zou voor het voorgezet onderwijs (VO) kunnen worden wat "realistisch rekenen" voor het PO is gebleken. Bij "realistische rekenen" werd alleen naar de uitkomst gekeken en niet naar de manier van oplossen. Bijgevolg krijgen leerlingen op de basisschool onvoldoende voorbereiding voor algebra op de middelbare school. Naar analogie hiervan kan WDA als volgt uitpakken: Als "denken" betekent dat je de som goed beantwoordt en we niet kijken naar hoe je aan de uitkomst komt ("allerlei aanpakken zijn mogelijk", dus "maar wat proberen" mag ook) dan raken wij zeer afhankelijk van goede som-ontwerpers die zorgen dat je voor iedere aanpak toch wel degelijk goed moet nadenken. Waarschijnlijk zijn de vraagstukken in het VO zo complex dat zulk een debakel als in het PO minder kans krijgt. Het bezwaar tegen WDA ligt eerder in de andere punten en de uitstraling naar het PO.

Weinigen lijken nu diepgaand naar de problematische kanten van WDA te kijken, wellicht omdat men ook niet goed weet wat men aanmoet met enerzijds uitleg " taxonomie Bloom", "1956 ..., niets nieuws onder de zon", en anderzijds de bewerking dat het "nieuw" is, of "eindelijk gaan we het ook daadwerkelijk doen".

Zoals gezegd, het is slechts mijn vermoeden en geen uitgewerkte diagnose. Wellicht meent men dat ik hiermee onrecht doe aan velen die juist hun best hebben gedaan om de WDA mooi vorm te geven. Zulk onrecht wil ik beslist niet doen. Ik adviseer sinds 2008 tot een parlementair onderzoek naar het onderwijs in wiskunde, en de introductie van WDA kan worden meegenomen. Doorstaan zowel het concept van WDA als de wijze van introductie (welke alternatieven zijn er bekeken ?) de kritische blik dan kunnen degenen die er hard aan gewerkt hebben met grotere voldoening van de waardering genieten, welke waardering nu diffuus is.