Proper definitions for uncertainty and risk

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Summary

The commonly adopted definitions of risk and uncertainty generate conceptual problems and inconsistencies, and they are a source of confusion in general. However, alternative and proper definitions are: (1) First there is the distinction between certainty and uncertainty. (2) Uncertainty forks into known (assumed) and unknown probabilities. (3) Ignorance, or unknown probabilities forks into known categories and unknown categories. (4) Known categories forks into 'including the uncertainties in the probabilities by explicitly assuming a uniform distribution' (Laplace) or neglect (or use other non-probabilistic techniques). Note that the term 'risk' has not been used in the 4 points above, so that an independent definition is possible. 'Risk' can be defined as the absolute value of probable loss, i.e. as (rho) $\rho = -E[X; X < 0]$. Also, relative risk is the probable loss with respect to a target t, giving $\rho(t) = t - E[X; X < t]$. The definitions provided here are directly in line with the Oxford English dictionary. It turns out that textbooks generally can keep their mathematics but will best rewrite their texts to these definitions. Not only the students and the general public will benefit from this sudden clarity, but eventually also statistics and economic theory themselves.

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Introduction

This discussion will present proper definitions for uncertainty and risk. Such definitions are required since the current definitions in common use are rather erroneous and generate conceptual problems.

Uncertainty

The new definitions are - see also Figure 1:

(1) First there is the distinction between *certainty* and *uncertainty.*

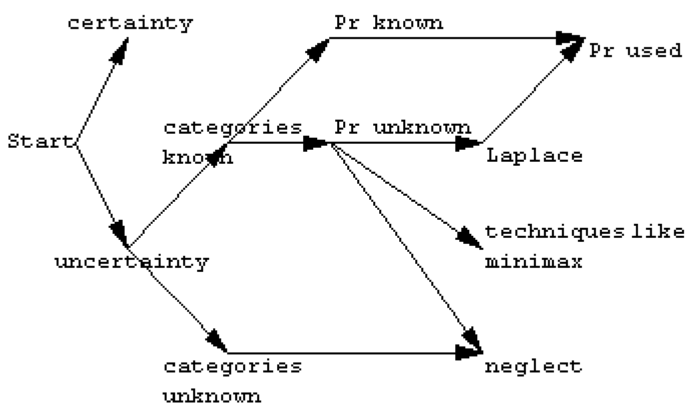
(2) Uncertainty forks into *known categories* and *unknown categories.*

(3) Known categories forks into *known* and *unknown probabilities.*

(4) Unknown probabilities forks into *assuming a uniform distribution* (Laplace) or use non-probabilistic *techniques like minimax* or *neglect.*

A.S. Hornby (1985) "Oxford Advanced Learner's Dictionary of Current English" defines 'uncertain' as: "1 changeable; not reliable: *~ weather; a man with an ~ temper.* 2 not certainly knowing or known: *be/feel ~ (about) what to do next; a woman of ~ age*, one whose age cannot be guessed". The above fits this.

Figure 1: A diagram of the new definitions



Note that these definitions only use certainty, knowledge and the distinction about categories (category-uncertainty), and that they do not use the term 'risk'. Thus an independent definition of 'risk' is possible.

Risk

Hornby (1985) defines 'risk' as: "(instance of) possibility or chance of meeting danger, suffering loss, injury, etc." Also: "at the ~ risk of / at ~ of, with the possibility of (loss etc.)".

Thus, if there are possible outcomes $O = \{o_1, o_2, ..., o_n\}$, then the situation is risky if at least one of the o's represents a loss. The risks are the o_i that are losses, thus $Risks[O] = \{o_i \in O \mid o_i$ is a loss$\}$. The risk factors are the positions or index numbers of the risky outcomes, the i's, or the dimensions (the causes that make such positions to be filled).

We will use the term 'valued risk' when a risk is valued with money or utility. When all risks have been made comparable by valuing them, then we can add them, and we will use the term *expected risk value* for the *expected value of the 'valued risks'*. Then, crucially, once these definitions are well understood, then we may also use *'the risk'* for the expected risk value. [2]

With such understanding, risk will be $\rho = -\text{E}_{X<0}[X]$ [3] or better $\rho = -\text{E}[X; X < 0]$. [4]

Valued risk deals with the cases when probabilities are known *or* when unknowns are assumed to be uniformly distributed over known categories. It is not customary to use the term 'risk' for unknown categories. For example, it is uncommon to say, or write economics papers about this, that "all our lives are *at risk* of a suddenly imploding universe, or black hole hitting Earth, or waking up as a cockroaches". Such real 'Acts of God' are commonly neglected. Note though that it still remains possible to say that a situation is risky even though one cannot put a number to it. Above expectation may be indeterminate since one may lack knowledge about the probability distribution or even the categories.

When the probabilities are subjective, then 'the' risk will be subjective too, in the same way as one can speak about 'the' expected value and 'the' spread even when these have been determined while using subjective probabilities. The definition of risk is an procedural one, since it simply adopts the procedure of how to determine an expected value (over a specified domain). Some colleagues wonder how it would be possible to arrive at 'the' risk, which sounds objective, when one starts out with subjective probabilities. Such a discussion quickly leads to the discussion about the foundations of probability theory and statistics. However, once the procedural nature of the definition of risk has been recognised, then this issue disappears. Different people with different subjective probabilities can calculate different risk values, and

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[2] Thus there is a subtle distinction between:
(A) *The risk,* that is single (i.e. non-plural), and gives the expected value of the valued risks
(B) *The risks,* that thus is plural and gives the list of the the o_i that are losses. For a single outcome, we would have the difference between o and $\{o\}$ (element and singleton). With a list of outcomes $O = \{o_1, o_2, ..., o_n\}$ we also have lists of prices $P = \{P_1, P_2, ..., P_n\}$, and probabilities $Pr = \{p_1, p_2, ..., p_n\}$, and a utility function u. (Continued next page.)

The money valued risks are $X = \{x_1, x_2, ..., x_n\} = O * P = \{ P_1\, o_1, , ..., P_n\, o_n\}$.

The utility valued risks are $U = \{ u(o_1), .., u(o_n)\}$. The expression $U^* = u(o_1, .., o_n)$ is less appropriate since the outcomes are mutually exclusive. However, since one might consider cases where one has some utility about 'the whole situation', the U^* might still be useful.

[3] Thus μ stands for the expected value and σ for the standard deviation (spread), and ρ the risk. Then, use R for the coefficient of correlation. Note that the use of 'spread' facilitates translation from learned journals to popular audiences that are less familiar with 'standard deviation'. Authors that use the word 'spread' for the difference between a futures and a spot price, should relabel to 'time premium'.

[4] The notation comes with a short story. I had considered it earlier myself, but had been afraid that the semi-colon would be confused with the conditional, and I therefor arrived at the notation $\rho = -\text{E}[X < 0]$ as used in an earlier version of this paper. In a personal discussion, Richard Gill (University Utrecht, KNAW) however found the latter mathematically improper, and came up with the same semi-colon notation. Andreas Kyprianou (UU as well) confirmed this. The common view now is, that once the distinction is made, there need be no confusion between the semi-colon as used here and the straight bar for the conditional. Another format is $\text{E}[X * I_{X<0}[X]]$ where $I_A[X]$ is the indicator function with value 1 if $X \in A$ and 0 else. This format no doubt increases definitory clarity, but the semi-colon shorthand is not bad and has the advantage of being short. Another option mentioned by Richard Gill is to use $\rho = -\text{E}[X^-]$, where X^- denotes the random variable with the negative values. This likely might be the most elegant notation, but it requires additional explanation of "X^-" while "$X < 0$" is quite self-evident, and while the latter allows a direct extension towards relative risk based upon "$X < t$".

the advantage of the common definition of risk is that everybody knows how these values been calculated, so that there is an advance in communication.

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Relative risk is defined as $\rho(t) = t$ - E[X; $X < t$] for some target level t. Risk (or *absolute* risk) takes $t = 0$, and relative risk would allow for a different target level.

An interesting application is when X is a stochastic rate of return and r the certain rate, so that there is relative risk $\rho(r) = r$ - E[X; $X < r$]. This relative risk answers the question: What is the probable loss with respect to a target return of r ? Here, $r - \rho(r)$ = E[X; $X < r$] gives the weight of underperformance in the total target return (which weight has to be compensated by probable profits to achieve the target).

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Conditional (relative) risk is defined as $\kappa(t) = t$ - E[$X \mid X < t$] for some target level t. With respect to rates of return, conditional risk $\kappa(r)$ answers the question: What would one expect to lose with respect to r, if earnings actually underperform and fall below r. Indeed, $r - \kappa(r)$ would give your expected return when actually underperforming.

Conditional risk is related to relative risk by the property that E[$X \mid X < t$] = E[$X < t$] / Pr[$X < t$]. The probable loss thus is corrected for the probability of the loss. Or, the probability measure in the expectation is corrected so that a density is taken that sums to 1. [5]

Example

In everyday parlance, profit and loss are nonnegative concepts. For example, if the difference between revenue and costs is \$-10, then your loss is \$10. It is only in mathematical economics that profits are defined as a general profit function such that 'negative profits' are possible. To understand risk, we however return to the everyday parlance convention.

Let us consider a *binary* prospect that can give *profit* with probability p, and *loss* with probability $1 - p$. We denote this as Prospect[*profit, -loss, p*]. We call *profit * p* 'probable profit' and *loss * (1 - p)* 'probable loss'. Then the following definitions apply:

- Expected Value = $\mu = p\ profit + (1 - p)\ (-loss)$ = probable profit - probable loss
- Risk = risk value = expected value of the risks = probable loss = $(1 - p)\ loss$
- Risk Ratio = Risk / (ExpectedValue + Risk) = $(1 - p)\ loss\ /\ (p\ profit)$
- Thus: Expected Value = $p\ profit$ (1 - Risk Ratio)
- Risk Probability = cumulative probability of all losses (in this case $1-p$)

Risk is the (absolute value of the) down side of a bet. A venture is judged to be risky if the probable loss is large. Note that this notion still is somewhat vague. A probable loss can be large because of the probability or because of the sum of money involved. This vagueness is unfortunate, in some respects, but here is little to be done about it, since this vagueness is inherent in working with probabilities. In fact, this vagueness is an essentially positive aspect of

[5] For (relative) risk we don't want to use the conditional distributions. For example, if there would be a small loss with a small probability p, the conditional might turn this in a large 'risk', since $1/p$ is would be a large number. So for risk we have a proper measure in the 'probable value' (loss * probability).
Risk is concerned with one's *worry* that bad information *might* arrive while it may not arrive. The conditional applies only if indeed new information arrives that the returns will remain below that target level. (Though the conditional might remain hypothetical.)

working with probabilities. For, when we have different prospects, then we can order and evaluate them on risk, neglecting differences in losses and probabilities.

A more general (non-binary) prospect can use lists. Consider for example the object w = Prospect[{10, -5, -2}, {0.4, 0.4, 0.2}], where the first list gives the possible outcomes in dollars and where the second list gives the associated probabilities. If we want to simplify the situation by collapsing or projecting w into a *binary* prospect, then it is most sensible to keep the risk and the risk probability constant, which gives v = Prospect[10, -4, 0.4]. The binary prospect v summarises more forcefully what we are up against, compared to the more complex list structure given by w. The prospects w and v are equivalent in terms of expected value and risk, but the spreads differ, namely $\sigma(w) = 6.95$ while $\sigma(v) = 6.86$.

One way to summarise this paper is to say that it helps us to understand how to aggregate risks. In general, we can define the 'risket' object as Risket[μ, σ, ρ, 1- p] where the $1 - p$ gives the 'risk probability' defined above, thus with the cumulated probability of all losses. This risket object is a useful tool to clarify thus two properties of aggregating risks: (1) The expected value, risk and risk probability should remain invariant, and the other aspects are derived. (2) Aggregation in this manner leads to loss of information on the spread. This kind of aggregation thus could be done when the spread has little relevance.

Wrong use in economics 1921-2000

The above definitions are proper in the sense that they conform to every day parlance and the definitions provided by Hornby's dictionary op. cit.. The definitions provided here however differ from the use within the economics literature. First there are the definitions of Knight 1921 that have been adopted widely in economics, as for example in The New Palgrave (1998:III:358). Or it has become custom in finance to associate risk with the spread. And some mathematical statisticians use another concept of risk. Let us discuss these in turn.

Uncertainty and risk

The New Palgrave, Eatwell c.s. (1998:III:358), gives the current common view:

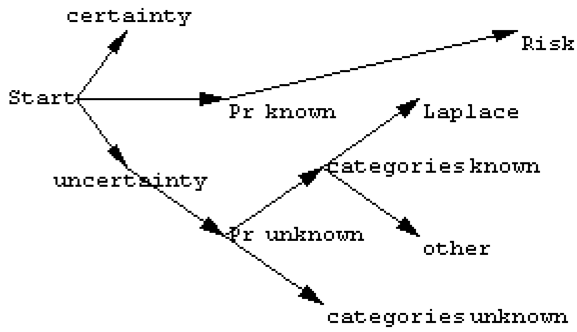
> "The most fundamental distinction in this branch of economic theory, due tot Knight (1921), is that of risk versus uncertainty. A situation is said to involve *risk* if the randomness facing an economic agent can be expressed in terms of specific numerical probabilities (these probabilities may either be objectively specified as with lottery tickets, or else reflect the individual's own subjective beliefs). On the other hand, situations where the agent cannot (or does not) assign actual probabilities to the alternative possible occurences are said to involve *uncertainty*."

Indeed, most economic texts use this distinction in this manner (at least, up to now). However, I cannot disagree more. The objections to Knight's concept are:

(a) Certainty and uncertainty are *binary*. So, if a situation is not uncertain, then we have certainty, and there is no assigning of probabilities.
(b) If I am uncertain about a situation and assign equal probabilities to all cases - the Laplace suggestion - then according to Knight this no longer is uncertainty!
(c) In Hornby's definition, the distinction is not between known and unknown probabilities, but the distinction is between *events* and *human thought*.

Figure 2 contains a diagram of the objectionable use of terms 1921-2000.

Figure 2: A diagram of the current but objectionable use of terms



The diagram clarifies the inconsistency with the binary character of certainty/uncertainty, the curious treatment of "Laplace", and the over-use of terms by introducing the term 'risk' where there already is the qualification that the probabilities are known.

Risk is not uncertainty

Michael Rothschild and Joseph Stiglitz (1970) *equate* risk and uncertainty. They ask the question: "When is a random variable *Y* 'more variable' than another random variable *X* ? (...) Throughout this paper we shall use the terms more variable, riskier en more uncertain synonomously." This however is not proper English, reduces the scope for communication, and destroys some useful words.

It may also be doubted whether one can find a single quantitative measure of 'more uncertain'. Uncertainty basically comes with a whole list of criteria and subcases. A *Y* can be 'more uncertain' than an *X* in a qualitative sense, when the categories of *Y* are unknown while they are for *X*. For quantitative cases, it could be a lexigraphic ordering, with the first criterion the range, and the second criterion for example the spread. The Rothschild & Stiglitz distinctions could find a place here. However, my focus is now on the risk.

Risk is not the variance

The finance literature often uses the term 'risk' for the variance or spread (standard deviation) of the distribution of the rates of return of investments. This would be an improper use of the term. Suppose that one has a very profitable venture without the possibility of a loss. Suppose that the rate of return of this venture has a large variance, from mildly profitable to highly profitable. Is this a risky venture ? No, not in the usual understanding of the term.

Malkiel (1999) provides a useful and entertaining picture, in lay terms, of modern investment theory. He notes: "Surely riskiness is not related to variance itself, the critics say. If the dispersion results from happy surprises (...) no investors in their right minds would call that risk." (p202). However, Malkiel argues that distributions would be symmetric (and apparently around zero), and then conforms with the academic convention (the Markowitz frontier) to equate risk and spread. As such, it gives a good discussion of modern investment theory, as intended. However, my answer to this remains that it is useful to distinguish risk and spread.

Risk is not the negative of expected revenue

In mathematical statistics, some authors, like Ferguson (1967), define 'risk' as 'expected loss'. However, it appears that they actually regard 'loss' as the negative of total returns (i.e. -revenue), so the definition used is -(*p profit* + *(1-p) (-loss)*), which is the negative of the expected value. This use of the term 'risk' is inappropriate. My proposal is to use the word "due" to stand for the negative of expected value, so that the standard statistical decision theory (with the game against nature) can be described as minimising *due*.

Note on Bernstein's "Against the gods"

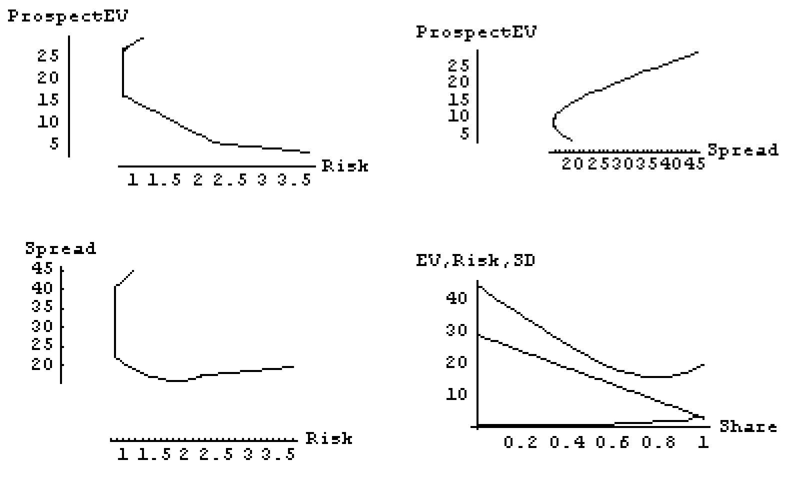
I came across Bernstein (1996) "Against the gods", and found it equally entertaining as his "Capital Ideas". It gives the great story of how humanity developed concepts to deal rationally with uncertainty and risk, i.e. the 'folly of the gods'. One comment is that Bernstein indeed emphasises Knight's and Keynes's statements on "uncertainty". My answer to that is, again, that unknown probabilities or even unknown categories indeed are serious cases of uncertainty, so that earlier writers on the subject were right in emphasising that seriousness. However, we should not be tempted to reserve the word "uncertainty" to only those cases. So with all due respect to Knight and Keynes, the definitions provided here are the proper ones.

Further consequences and developments

Cool (1999, 1999a, 2001a) further develops these notions for simple binary prospects, multidimensional prospects, joint prospects, and continuous probability densities.

An interesting application is a Markowitz-type efficiency frontier with risk rather than the spread. Figure 3 has been taken from Cool (2001a). It depicts the portfolio of two prospects *A* and *B*, with the share *s* of *A* in the whole portfolio running from 0 till 1. The statistics μ, σ, and ρ all depend upon this share *s*. The upper right hand corner gives the standard Markowitz efficiency frontier, which is familiar to the finance community. The other plots are new. We may assume that an investment advisor refers to the CAPM and then advises the client correctly. However, if the client basically is worried about risk, then there may be a communication confusion when the advisor substitutes volatility (historical spread) for the risk as defined here (and closest to the client's real worries). In this case, *A* turns out to be riskiest, even though it has a low spread. A risk averse client would opt for *B*, which also has a high expected value. *B's* spread is large, but the variation is in the high positive values, and there is little risk involved.

Figure 3: Markowitz efficiency frontier



Cool (2001b) further relates to 'expected utility theory' and develops an alternative for the latter. An explicit incorporation of risk, as defined here, seems better than the Arrow-Pratt measure for 'risk aversion' based upon the second derivative. It appears that the Allais paradox, subsequently, can be understood in terms of the wealth effect.

Interestingly, Rabin (2000) also appears to conclude: "Expected-utility theory is an utterly implausible explanation for appreciable risk aversion over modest stakes". A recent review of course is Starmer (2000). Expected Utility obviously already fails, as it is one-dimensional, compared to the two-dimensional Markowitz efficiency frontier analysis. Inclusion of the explicit risk measure should even work better.

Some competing concepts

The literature shows some other approaches to risk that can be usefully reflected upon.

(a) Value at Risk (VAR)

Value at Risk (VAR) is the expected minimum loss in currency units for some level of probability. For example, given a (one sided) confidence level of say 99%, all possible revenues should be higher than the value at risk.

The VAR thus still allows that with 1% probability a huge loss can occur. There is no weighing of the losses with the probabilities. This thus implies limited information. Also, the target t appears variable, so that when comparing different investments while using the same confidence level, different levels of VAR are compared - with unknown risk.

(b) Probability of ruin

Grandell (1991:1) discusses the traditional approach to collective risk theory, which concerns the model of an insurance company where the probability of ruin $\Psi(u)$ is defined as the probability that the cumulated net surplus of premium income minus claims will ever be below some u, e.g. eat up some initial capital $u = k$. Thus, there is a risk $\rho = u \,.\, \Psi(u)$, as the owners of the insurance company would lose u with the associated probability of ruin $\Psi(u)$. However, even if one is not ruined, there still is the possibility that the venture would be insufficiently profitable. In that case u would become time-dependent $u = k(0)\,(1 + \pi)^T$ with π the target rate of profit. The loss would then be defined with respect to this u.

(c) Relative risk in epidemiology

In epidemiology, subjects who are exposed to some agent like a virus or so can be expected to have a higher incidence of getting a disease than those who have not been exposed. Thus there is an 'incidence multiplier from exposure' IM, given as the ratio of conditional probabilities (exposed vs. non-exposed). Kelder (2001) gives the example in Table 1, where the incidence multiplier (of getting the disease from exposure) IM = (14 / 23) / (49 / 198) = 2.46.

Table 1: Epidemiology

# subjects	Incidence	No incidence	Total
Exposed	14	9	23
Non-Exposed	49	149	198
Total	63	158	221

Now it turns out, for epidemiology, that the term 'risk' is used merely for the (conditional) *probability* of a loss (disease, death), and that the term 'relative risk' RR is used for the ratio, i.e. the incidence multiplier IM. In fact, I have adapted Kelder's example, by using IM where he writes RR. The terminology in epidemiology may make sense when only one disease is being regarded, so that the loss is taken as fixed. When there are more diseases - or degrees of disease and types of subjects - then one would expect some weighing of the losses, and then the epidemological terminology breaks down and interferes with standard parlance. Indeed, common parlance may allow a person to use the term 'risk' also for a single disease, but this in itself has little implication for the general use. A question by a lay person like "What is the risk that I get the disease?" seems to have its best answer in "The probability is 14/221, or between 49/198 and 14/23. Tell me whether you have been exposed and I can be more specific. Tell me how bad you think your loss will be, and I will tell your risk." Thus, overall, I would rather use the term 'incidence multiplier (from exposure)' as done above, rather than 'relative risk' - though of course one should be modest in one's views and hopes concerning the terminology used in specific fields of research.

Ellsberg's Paradox

Epstein (2001) relates the paradox created by Daniel Ellsberg: "In a version of the Ellsberg Paradox, the decision-maker is confronted with two urns, each containing 100 balls that are either Red or Blue. She is told that there are 50 of each color in the first ("unambiguous") urn, but no further information is provided about the second ("ambiguous") urn. There is a widely exhibited preference to bet on drawing Red (or Blue) from the first urn rather than from the second. Though such rankings are intuitive, they are inconsistent with subjective expected utility theory and, more generally, with reliance on *any* single probability measure to represent beliefs. Thus the paradox illustrates the behavioral meaning of the Knightian distinction between risk (measurable or probabilistic uncertainty) and ambiguity (unmeasurable uncertainty)."

As 'measurable' and 'unmeasurable' are polar opposites, Epstein uses polar opposites 'risk' vs 'ambiguity'. I would not do that since it destroys some useful words. It is better to use 'ambiguity' for 'different meanings at the same time'. Also note that Epsteins example has known categories, and that the problem is that the probabilities are unknown - with is Laplace and not Knight.

In itself, the Ellsberg paradox suggests that de Laplace solution of assuming equal probabilities would be too simple. Namely, consider x = Prospect[a, b, p] with unknown p. Then $\mu = \mathrm{E}[x] = p$

$a + (1 - p)\ b$, and if p is uniformly distributed and if we are allowed to switch the integrals, then $E[\mu] = \int_0^1 \mu \ dp = ½\ (a+b)$, which means that we might as well assume that p = ½.. This also appears to hold for the risk defined above, with ρ = - ½ b here. On first sight, thus, people are plain irrational when they are not indifferent between the urns. However, given the observation that people are not indifferent between the urns, we could wonder about the adequacy of that assumption of the uniform distribution. Properly, *any* distribution is possible, and the Laplace assumption may only apply to the equal likelihood of any distribution. This line of thought leads to considering the work of Newcomb 1881, Benford 1938, Pinkham 1961, Nigrini 1992 and Hill 1996, on the 'distribution of distributions'. If d are the first digits of p, then their relative frequency would be $f(d) = {}^{10}\text{Log}(1 + 1/d)$. Since this would hold for the probabilities of both winning and losing, and we don't know which is what, we could take the average of these relative frequencies ½ $(f(d) + f(10^n - d))$, where n is the number of digits in d. [6] The consequence is that extreme values like 0.1 and 0.9 have a higher probability than values around 0.5, which means that the risk, as properly defined above, is higher - even though the expected values for both urns would be the same. In my impression, this solves the Ellsberg paradox.

Other works

This discussion has been adapted from chapter 35 of Cool (2000) while *Mathematica* programs are available in Cool (1999, 2001a) and a longer discussion is available as Cool (1999a). I benefitted much from Luenberger (1998) and Tajuddin (1996).

Conclusion

Words are flexible, and their meaning need not be fixed. Scientists can adapt the meanings of words to their needs, as long as everybody understands what they are talking about, in their branch of research. The problem with the use of the word 'risk' in statistics and economics is that the concepts and words are very basic, are also used in common language and for the communication with decision makers and the general public, and that there is some confusion, so that this flexibility of words leads to conceptual errors. By defining and fixing the concepts as suggested above, and by sticking to the meaning in the natural language, much clarity is gained. It turns out that textbooks generally can keep their mathematics but will best rewrite their texts to these definitions. Not only the students and the general public will benefit from this sudden clarity, but eventually also statistics and economic theory themselves.

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[6] For example, if p would be 0.125 and we would look at the first digits only, so that $n = 1$, then it starts with digit 1 (always neglecting 0). Then $q = 1 - p = 0.875$, and this starts with digit 8 instead of 10 - 1 = 9. However, the frequencies would be compensated by cases like q = 1 - 0.0125 = 0.9875 etcetera.

Literature

Bernstein, P. (1992), "Capital ideas", The Free Press

Bernstein, P. (1996), "Against the gods", Wiley

Cool, Th. (1999, 2001a), "The Economics Pack, Applications for *Mathematica*", Scheveningen, JEL-99-0820, ISBN 90-804774-1-9, see www.gopher.nl

Cool, Th. (1999a), "Proper definitions for risk and uncertainty", EconWPA ewp-get/9902002, Internet Economics Working Papers Archive

Cool, Th. (2000), "Definition & Reality in the General Theory of Political Economy," Samuel van Houten Genootschap, The Hague, see www.gopher.nl

Cool, Th. (2001b), "Voting Theory for Democracy", Scheveningen, ISBN 90-804774-3-5, see www.gopher.nl

Eatwell, J. c.s. (1998), "The New Palgrave", Macmillan (entries risk, risk aversion, CAPM)

Epstein, L. (2001), "Sharing ambiguity", AER Vol 91 No 2, May

Ferguson, Th. (1967), "Mathematical statistics", Academic Press

Grandell, J. (1991), "Aspects of risk theory", Springer

Hornby, A.S. (1985) "Oxford Advanced Learner's Dictionary of Current English", Oxford

Kelder, S. (2001), "If it's not fun", www.sphnt.uth.tmc.edu/de/ph2610/2lecture/index.htm

Luenberger, D. (1998), "Investment science", Oxford

Malkiel, B.G. (1999), "A random walk down Wall Street", Norton

Rabin, M. (2000), "Risk aversion and expected-utility theory: A calibration approach", Econometrica Vol 68, no 5, September, p1281-1292. The reference and quote have been taken from from Economic Intuition, Fall 2000, www.economicintuition.com

Rothschild, M., and J. Stiglitz (1970), "Increasing Risk: A Definition", J. of Ec. Theory, Vol 2 No 3, September

Starmer, C. (2000), "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk", JEL Vol XXXVIII (June) pp 332-382

Tajuddin (1996), "A simple measure of skewness", Statistica Neerlandica pp 362-366