Reading Notes on Voting Theory for Democracy (VTFD)

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Abstract

This document concerns Colignatus (2001, 2014), *Voting Theory for Democracy* (VTFD) (online). In the course of time, there may be some comments, by either readers or myself. Some comments might eventually cause some adaptation in a new edition of the book. It appears useful to collect such comments in this separate document, that can be put on the book's website for the benefit of readers (and as reminder for myself). A comment here does not necessarily invalidate VTFD. Chapter 1 of VTFD distinguishes new and advanced readers. Both might have comments deriving from some mismatch, e.g. a new reader going too fast or an advanced reader skipping too much of the re-engineering in the earlier chapters. These Reading Notes for now only concern VTFD Sections 9.1 & 9.2.

1. Introduction

See the Abstract.

2. VTFD 9.1 & 9.2 on Arrow's Impossibility Theorem

VTFD Section 9.1 provides the technical definition of a Social Welfare Function Generating Mechanism (SWF-GM).

VTFD Section 9.2 originated as Colignatus (1992) (paywall), later edited into Colignatus (1997) (online).

There are sheets (2018), presented at workshop 18 of the Dutch & Flemish annual conference, June 7-8 2018 in Leiden.

Colignatus (2017) discusses an 2012 interview with Arrow by Aaron Hamlin.

VTFD Section 9.2 uses some deontic logic. VTFD Section 3.5 (p52-57) therefor has an introduction into deontic logic and its link to common preference (or the utility function). This introduction is quite limited and targeted at what VTFD calls the "Deontic Axiom" (DA): (*Op* & (*p* => *q*)) => *Oq*.

- A better format could be (*Op* & |-(*p* => *q*)) => *Oq* but VTFD has no development of logic, and VTFD simply states that *p* => *q* must be a theorem and not some contingency.
- VTFD embeds morality and mere preferences and thus finds that this DA must hold.
- Within the scope of VTFD the name DA is used, since VTFD uses little else.
- Within deontic logic this DA is called "RM" ("Rule of Monotonicity"), see McNamara (2014) or "necessitation" (Hansson (2004:5). In "Standard Deontic Logic" (SDL) [1] this is a theorem and no axiom. Within the community of deontic logicians SDL is a bone of contention, but it is still called SDL.
- I thank an author of deontic logic for helping out in getting some recent references for these Reading Notes of 2018. I did not think that this was necessary before but given the present comments it apparently is.
- The reader thus is warned that deontic logic is no clear-cut case like propositional logic, and that there still is much discussion, and that for VTFD it suffices to take DA as the particular aspect that derives from embedding morals and mere preferences. Standard textbooks on logic tend to extend into modal logic, potentially under influence of physics. I deem it of critical relevance for Political Economy that VTFD provides this extension into deontic logic. (See also Hansson (2004) for a look at such embedding in general preferences – though I would have some questions on that paper.) [2]

Schulze (2011) in a supposed "review" called section 9.2 "mumbo-jumbo". It may well be that Schulze (at that time) wasn't familiar with deontic logic. See Colignatus (2013) for a protest that this is not the proper way of doing a review. A reviewer must specify what would not be understood and should consider to contact the author for a clarification. It is a false presumption that a reviewer would know all and might not be in position to ask questions. A review is at the service of the readership, and not intended to create more confusion.

These Reading Notes on sections 9.1 & 9.2 derive from comments on content by a student of deontic logic, anonimised as XYZ. This student thus didn't treat these sections as "mumbo-jumbo" but looked at the content. However, it appears that this student didn't read precisely, and he might be lacking in mathematical training. See Chapter 1 of VTFD for the target audience. Much too frequently, this student actually misrepresents what VTFD does. This causes distraction and needless discussion. I provided this student with the answers mentioned below. The student however was not willing to withdraw the misrepresentations. It is okay to have different views about clarity, but misrepresentation means disinforming other

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[1] Wikipedia is a portal and no source: SDL: https://en.wikipedia.org/wiki/Deontic_logic#Standard_deontic_logic

[2] It may also be observed that philosophers may have little background in mathematics. A more general warning is given by Colignatus (2009, 2015). Mathematicians are trained for abstract thought and thus miss out on the training for empirical science. Education in mathematics is an empirical enterprise. One can imagine that something goes wrong here. In the same way a training in philosophy is targeted at abstraction. Philosophers may get lost in abstraction too. Giving them an additional training on mathematics only would not be sufficient. Thus the advice is that philosophers also get a training in an empirical science. A recommended field is the research in didactics of mathematics. This contains both abstraction and empirics, and would train aspiring philosophers on the empirics of studying how to teach and learn abstraction.

people about what VTFD does. Also when the student would be silent on VFTD then this still would be a misrepresentation, for then he withholds information about VTFD in explicit reference that would be open to public discussion. Thus my conclusion is that this student is in breach of scientific integrity (or what legal courts might label as a "hostile witness" [3]). I am sorry to report this, but check the comments and answers below. PM. As a general statement, the student wrote to me (paraphrased): "I have read the two sections now. You write well, and the discussion of the relevance of Arrow's Theorem is informative. Unfortunately, the presentation still rambles too much to be convincing. Below are the problems that I had w.r.t. technical matters, content and style." So let us see what would be "rambling", and let us wonder why the student would not want to correct given my answers to his imprecise reading.

3. VTFD 9.1 & 9.2: On style

3.1. Notation 1

Student of deontic logic XYZ wrote (paraphrased): "In notation and definitions you are unorthodox while there is not always a reason for it. If there are no compelling reasons to do otherwise, I would simply follow the conventions and also follow the standard notation for preference orderings. Stylistic stubbornness can mean loss of readers, and that would be a pity."

Answer: VTFD provides an elegant answer to its conditions:

(a) Part of the notation is dictated by the wish to integrate utility and morality (deontic logic)
(b) Part is dictated by the use of *Mathematica*
(c) Part is dictated by the need to re-engineer voting theory from the bottom up.

The deviation from conventional notation and definition is minimal given those conditions. Apparently, the student only read VTFD 9.1 & 9.2 and not Chapter 1 of VTFD. I did invite the student to look at VTFD 9.1 & 9.2 but I presumed he wouldn't overlook the general setting of the book (or would be spurred to look at it when questions would arise). The student lacks the scientific discipline to check what he is commenting about.

3.2. Notation 2

Student of deontic logic XYZ wrote (paraphrased): "VTFD:246, 3rd line: I do not understand your notation: the use of symbols from set theory is incorrect here." (This concerns: {{A, B, C}, {A, C, B}, {A, B, C}}.)

Answer: VTFD has been written in the environment of *Mathematica.* In *Mathematica*, lists are denoted with curly brackets (accolade). These brackets can attribute Orderless (sets) or Ordered (vector). The latter applies in this case. [4]

3.3. Judging work by others

Student of deontic logic XYZ wrote (paraphrased): "You are sometimes too quick in your judgments about the work by others - the charity principle is a great asset in science."

Answer: I take time and effort to study the work by others, and provide substantiated criticism with respect to specified statements by others. I also aspire at clarity what *precisely* would be erroneous or a cause for confusion. There has been much confusion about Arrow's Theorem and Arrow's interpretation so that this precision must be valued highly. Please to do not confuse precision and lack of charity. Please do not confuse authors and texts.

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[3] Wikipedia (a portal and no source): https://en.wikipedia.org/wiki/Hostile_witness
[4] http://reference.wolfram.com/language/howto/WorkWithLists.html

Please do not confuse lack of precision with deliberate misrepresentation either. However, when someone is given precision and then still refuses to correct, then the question arises whether there are ulterior motives (like not wishing to admit that the comment was erroneous, and should have been an informative question anyway rather than presented as a criticism).

3.4. Journals and book publishers

Student of deontic logic XYZ wrote (paraphrased): "If you want reactions from the academic world, simply send an article to a journal or a book to a book publisher."

Answer:

(a) VTFD already has been published. I have sent it in for a review. See Colignatus (2013) about malpractice in the academic world w.r.t. such review. See such also for another book "Conquest of the Plane". [5]

(b) The academic world confuses peer review with commercial publishing. The only thing that is required for academics is to do peer review, and they can put papers & books and their reviews in a open access database. The entourage of commercial publishing is a distraction, it strips authors from copyrights and puts papers behind paywalls, which means that this approach is in contradiction with open access publishing. [6]

(c) Thus, for this student, the crucial question is: when your comments have been answered, so that VTFD 9.1 & 9.2 should be clear to you, would you be willing to provide a peer review statement that the analysis in these sections has been corroborated ? If you are not willing to incorporate the answers in your evaluation, what is the problem ?

(d) It is quite curious that student XYZ makes this comment to go to journals and book publishers. Why avoid taking a position and why refer to others ? (Some academics might hide behind such paywalls. If people will not pay to see what errors they write, then those errors will not quickly be exposed, and those errors might even affect an entirely line of "education and research". Observe that Colignatus (1997) has been online for some 20 years now, and apparently has been neglected by researchers on Arrow's Theorem, see Colignatus (2017).)

4. VTFD 9.1 & 9.2: Technically

4.1. Simpler ?

Student of deontic logic XYZ wrote (paraphrased): "Would it not be possible to simplify your deontic argument ? Namely: (a) Assumption: Ought implies Can, (b) Arrow shows that not all conditions can be satisfied by an SWF-GM (whence not-Can), (c) Conclusion: We should not demand that a SWF-GM meets all his conditions."

First some information:

- VTFD:241 rule 2: "2. Similarly, one cannot be morally obligated to a logical impossibility"
- In deontic logic a common formalisation is $Op => \Diamond p$. Wikipedia (portal, no source): "the Kantian claim that "ought implies can"" [7] VTFD however doesn't have modal logic.
- VTFD:247: "First proof by realism (AF) ...."
- VTFD:247 Lemma I, discussion: "We already concluded that the most preferred point (Op) would also be the chosen point (p)."

Then the answer:

(a) For a part such summary holds. But VTFD 9.1 & 9.2 do much more. From the contradiction itself it is immediately obvious that not all conditions can be met. This is a simple

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[5] http://thomascool.eu/Papers/COTP/LOWI/Index.html

[6] https://boycottholland.wordpress.com/2016/10/12/letter-to-vsnu-and-others-on-membership-dues-and-open-access-publishing/

[7] https://en.wikipedia.org/wiki/Deontic_logic

finding that doesn't constitute a new contribution. The new insight from VTFD 9.1 & 9.2 is that Arrow's claims on reasonableness and moral desirability cannot hold. Thus this proposed "simplification" misses the point of VTFD 9.2.

(b) I found "Ought implies possible" a bit too abstract for the readership at this point, requiring the need of more discussion of both deontic logic and modal logic. Thus rule 2 quoted above suffices, and then Lemma I gives both the Kantian rule specified for the SWF-GM and that when an optimum is possible, then collective choice results into that optimum. (The argument might only seem "simpler" when you have background in modal logic to draw from.)

(c) VTFD 9.2 has also other proofs that reject Arrow's Moral Claim (AMC). Perhaps the abundance of possibilities for rejection of Arrow's misinterpretation is dazzling, but I deemed it useful to list the main ones. (The sheets (2018) provide an overview as well.)

4.2. Definition of the deontic axiom (DA) in VTFD

QUOTE VTFD 243

Deontic Axiom (DA): For all p, q: $(Op \ \& \ (p => q)) => Oq$

There is some discussion between moral theorists whether DA really holds. It may be felt that the logic is not very compelling for empirical relations of dubious causality. However, if p => q reflects a logial truth, then DA is commonly accepted.

UNQUOTE

See also Section 2 above that Sections 9.1 & 9.2 must be read in the context of VTFD and cannot be read entirely by themselves.

4.3. Comment on DA in VTFD

Student of deontic logic XYZ wrote (paraphrased): "The deontic axiom (DA) as it is now formulated (DA) is certainly not an axiom in standard deontic logic (SDL) and is not derivable in it. As far as I know there is also no discussion about the axiom, as you write ("There is some discussion between moral theorists if DA really holds") because it is so evidently implausible in this form, and if there is discussion, then give the references. What is deductible in SDL, and that you also indicate in axiom, is an adaptation of DA in which $p => q$ is a theorem or in which the implication is replaced by $O(p => q)$. Anyway, formulate DA properly."

Answer (using quotes for new paraphrases):

(1) "There are no references to such "discussion"." Answer: True. In 1992 it was fairly obvious that there was such discussion. In 2018 it might perhaps be useful to specify references. See Section 2 above, that might be included in VTFD Section 3.5. (It is remarkable that deontic students who would know the references apparently are most in need of references.)
(2) "The form with $p => q$ definitely is no axiom in deontic logic." Answer: True for current systems like Mally, Von Wright or SDL. The name "Deontic Axiom" has only meaning within VTFD. See Section 2 above. However, anything might be taken as an axiom, and one cannot object to my adoption of DA as an axiom within the confines of VTFD.
(3) "The form with $p => q$ is not derivable in SDL." Answer: I haven't tried and this seems true. However it is derivable in Mally's system, see Lokhorst (2013), theorem (6).
(4) "The form with $p => q$ is evidently implausible. There really is no discussion about it. If there would be discussion, then provide references." Answer: There was the discussion in going from Mally 1926 to current SDL. Perhaps Mally's form currently is seen as "evidently implausible" but basically this still would need explanation: and this explanation would be a form of discussion by itself. Lokhorst (2013): "It is only a small step, not a giant leap, from Mally's system to modern systems of deontic logic, so Mally's pioneering

effort deserves rehabilitation rather than contempt." Thus, VTFD is evidently sensible in making the distinction between the two forms and indicating that there is discussion.

(5) "If you say that $p => q$ is a theorem, then you should write |- ($p => q$)." Answer: VTFD is no book on logic and for the readership it is distractive to introduce more notation than necessary. It suffices to state that we can only employ DA when $p => q$ is a theorem.

(6) "The form would also be a theorem for $O(p => q)$, logically stronger but more elegant than adding the assumption that the implication should be a theorem, and in Op & $O(p => q)$ you can refer to discussions in deontic logic, for $O(p => q)$ is defended as the correct formalization of conditional duties." Answer: VTFD has no need for this form. (It is actually doubtful whether this form is so useful for economics.)

(7) Thus the student's comment consists of half-truths and misrepresents VTFD by suggesting that something would be incorrect.

4.4. Theorem A.1

Student of deontic logic XYZ wrote (paraphrased): "In the first two proofs of Theorem A.1, you apply DA to propositions $p => q$ that are *not* a theorem. This is at odds with your interpretation of DA."

Answer: See VTFD:242 that $a => \neg a$ and $a =>$ *falsum* are forms of Arrow's Theorem. Arrow's Impossibility Theorem definitely is a theorem.

Perhaps this student of deontic logic wants to read |- $a => \neg a$ and |- $a =>$ *falsum* ? But VTFD avoids such formalism when it can be avoided.

4.5. Budget set and Lemma I: information

VTFD 243: "AF feasibility, X is the budget set (rather than the whole space)"

VTFD 243: "I thus agree with Arrow's 1950 statement: "My own feeling is that tastes for unattainable alternatives should have nothing to do with the decision among the attainable ones; desires in conflict with reality are not entitled to consideration." Thus, also, when one point is (socially) most preferred, it is the one consumed."

Lemma I: AF implies that a SWF-GM p satisfies the property $Op => p$

4.6. Budget set and Lemma I: comment 1 & its answer

Student of deontic logic XYZ wrote (paraphrased): "The proof is incomplete because AF has not been given a formal definition (page 243). Why does the budget set restriction imply that "AF means that desires (Op) in conflict with reality are not entitled to consideration ($\neg p$) ?" This does not follow on formal grounds."

Answer:

Section 9.1 p236 *explains formally* that X is the commodity domain and that budget B is a subset of X.

It is economics: If p falls outside of your budget you cannot choose it, whence $\neg p$.

Within this formalism, condition AF is a direct interpretation of Arrow's statement: "desires in conflict with reality are not entitled to consideration". AF means that we do not aggregate over X but over some budget. All this in 9.2 is embedded in the formalism of 9.1.

4.7. Budget set and Lemma I: comment 2 & its answer

Student of deontic logic XYZ (paraphrased): "In your deduction of "$Op => p$" you seem to work with a Can operator, $Op => Can\ p$. But that is missing in the lemma and you have not

said anything about the logic of that operator or the relationship with AF to draw conclusions about it."

Answer: There is no need to discuss $Op => \Diamond p,$ namely, see the argument on a "simpler" version.

4.8. Budget set and Lemma I: comment 3 & its answer

Student of deontic logic XYZ (paraphrased): "Analogous to DA, Lemma I can be valid if, like $p => q$ in DA, it is assumed that p is a theorem, but in that case the lemma would simply be p."

Answer:

(a) In DA, we use that $p => q$ is a theorem, not that p is a theorem.

(b) Perhaps this student wants to present another lemma, but Lemma I stands as it is. You cannot criticise a lemma by saying that there is another lemma.

4.9. Budget set and Lemma I: comment 4 & its answer

Student of deontic logic XYZ wrote (paraphrased): "There is no deontic logician who would accept Lemma I in this form. However, you use it for your proof of the Theorem A.1. This evokes the possible defense that it is not so much AMC that must be rejected, but the condition AF. The lemma is about a property of p that is defined in terms of p. This is unnecessarily confusing. Therefore delete "the property" in the lemma so that it becomes something like: "AF implies that for any SWF-GM p: $Op => p$.""

Also: "How can a proposition in which p occurs be a property of p itself?"

Answer:

(a) Indeed, the lemma survives when "the property" is dropped. When I included this term, I thought that it would be more agreeable reading, to also have some text elements rather than only formulas. We can now refer to "the property in Lemma I" for example. The reader may check other uses of the term "property" in VTFD. [8] It is a matter of empirics though. If more students find the term "the property" confusing here then it can be dropped for later editions.

(b) Perhaps this student of deontic logic XYZ thinks that properties must have a particular format ? VTFD is no book on logic and has no development of a theory on properties. Obviously VTFD is embedded in common usage. In common usage, the term "the property" need not be confusing. Consider the properties for numbers x: $Q[x] = \{x \mid x \text{ is an integer}\}$ and $Q2[x] = \{x \mid x / 2 \text{ is an integer}\}$. Then $Even[x] = \{x \mid x \text{ is an integer and } x / 2 \text{ is an integer}\}$. We can paraphrase this as "An even integer x satisfies the property that $x / 2$ is an integer." This has the same format as the Lemma here. What is confusing about "a property of p that is defined in terms of p" ? What confusion would you make, and why would you want to make that confusion when you know that it is a confusion ?

(c) Well, the lemma is as it is. Do you mean that you would not accept its proof ? Why not ? What is wrong with the proof ?

(d) Lemma I is not used in the proof of theorem A.1.

(e) Perhaps we must regard the comment as deriving from philosophy on the notion of self-reference ? A proposition is a state of nature. A sentence is the human expression of a proposition. Human expression requires the use of concepts. Those concepts are part of nature when one accepts that thought is part of nature as well. Concepts are relevant for

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[8] For example VTFD:258: "(...) we should make a proper distinction between a property that is universal and a property that is dependent upon the situation."

human understanding, but can be replaced by simpler versions again when we describe nature in observational terms. A SWF-GM is a concept (here embedded in a proposition) and thus has been formulated in thought, but in observational terms we see how people arrive at decisions. Humans develop concepts that imply a notion of "self-reference". One might consider that the concept of a SWF-GM comes with properties that involve a notion of self-reference. This need not imply that events in nature have such self-reference, as e.g. a circle does not think itself, and thus need not refer to itself.

4.10.   Restatement in section 9.2.3

VTFD:245: "We can use Condorcet's case to give a short proof of Arrow's Theorem, restricting our attention to majority voting."

The reader is advised to read well. VTFD states that it restricts its attention here, for the purposes of this short proof, to majority voting. This inserts an additional condition of majority voting which condition Arrow's Theorem doesn't have. VTFD has no claim that it would prove Arrow's Theorem in general in this manner. Also there is no need in VTFD to repeat this general proof. For 9.2 this restricted proof suffices, since most people would tend to think that procedures that do not satisfy majority voting would be less democratic. (In itself this could be wrong, since Borda may generate results that differ from the majority on the first preferences.)

Student of deontic logic XYZ wrote (paraphrased): "Section 9.2.3 is called "Restatement of Arrow's Theorem". Formally this section does not contain a restatement, and your analysis is not a "short proof" of Arrow either. You can say, as in many introductions to Social Choice Theory that the voting paradox *illustrates* that the majority rule also does not meet Arrow's conditions. But you do not use the majority rule because you say that in the example there should be indifference at group level, which assumption comes out of the blue and is an extra condition / axiom that you do not find with Arrow. For those reasons you cannot say that you give an alternative proof of Arrow's position (whether or not limited to a subset of SWFs).

Also by XYZ: "Your "Condorcet-proof" of Arrow (VTFD:245-6) is incorrect because you use an extra axiom (namely aMb (= there is a strict majority that prefers a above b), bMc, cMa => indifference on a, b and c). Arrow does not have such a condition and you do. Thus you do not provide proof of Arrow's position but of the impossibility of Arrow's conditions * plus * an extra condition."

Answer:

(a) The name of VTFD Section 9.2.3 on page 244 is only a name. Arrow's theorem is actually restated in 9.2.2 on page 242. Indeed 9.2.3 focuses on the proof rather than the theorem. VTFD makes a distinction between the logic of Arrow's theorem and its proof (that VTFD accepts) and Arrow's interpretation (that VTFD does not accept). Thus it is helpful to show why the proof can be accepted. There is no need to repeat the proper proof, that has been restated in many places, so this section has a different approach. From the setup of VTFD 9.2.3 it is obvious that the section only illustrates Arrow's theorem and proof, and the section is precise in mentioning the additional restriction: "We can use Condorcet's case to give a short proof of Arrow's Theorem, restricting our attention to majority voting." Thus the student only describes what VTFD does, and I do not understand why the student thinks that it would be a criticsm to repeat what VTFD does. (Remember his term "rambling".)

(b) A general theorem "for all x, Property[x]" can be refuted by showing a counterexample not-Property[$x0$]. VTFD 9.2.3 adopts this method. Arrow's conditions are insufficient for a choice method (they are actually inconsistent). Thus one must introduce additional assumptions to actually get a method of choice. Majority voting is a common notion for democracy, and thus this section shows that this already generates a problem.

(c) There is the common confusion between voting and deciding. There is the common neglect of the condition of the status quo. VTFD 9.2.3 clarifies those aspects as well. It is not clear why the student thinks that such clarifications "come out of the blue" when the issues are on the table and the clarifications are relevant.

(d) VTFD 9.2.10 Conclusion (VTFD:251) has a phrase "a proof of Arrow's Theorem that shows clearly the abuse by APDM". The emphasis of this statement is on the abuse of APDM. This conclusion has a clear dependence upon VTFD 9.2.3 with its explicit requirement on also using majority voting. This cannot be construed as a claim of an alternative proof of Arrow's theorem per se (as the body of the text doesn't support this). Perhaps though, to protect this conclusion from such abusive reading, this phrase must be adapted. [9]

(e) The statement by the student "you cannot say that you give an alternative proof of Arrow's position" gives a **misrepresentation** of VTFD, that there would be such a claim of giving an "alternative proof of Arrow's position". There is only a claim w.r.t. highlighting the role of APDM. (And it boggles the mind that the student is not satisfied with the clarification of his misunderstanding, and that VTFD would require a new edition with a revised text before the student might consider to look at the argument to be able to state that the analysis has been corroborated.)

5. VTFD 9.1 & 9.2: On content

5.1. Relevance of 9.2.7

VTFD:248 "Basically though, scientists can only advise on preferences, and the proper decision is up to the body politic."

Student of deontic logic XYZ wrote (paraphrased): "I have not yet seen the relevance of 9.2.7 for rejection of APDM / IIA - after all, the antecedent of the implication has not yet been demonstrated."

Answer:

(a) The relevance of this section is that it highlights the logical structure.
(b) The relevance is that conditions AU and AWP are identified as parts of the definition of a SWF-GM.
(c) The relevance of this section is that it highlights arguments for rejection of APDM.
(d) The relevance is that it is repeated that science can only advise on preferences, and the proper decision is up to the body politic.

5.2. Mueller's adoption of cardinality

Student of deontic logic XYZ wrote (paraphrased): "I have doubts about your certainty in your rejection of Mueller's view. How can he draw a "wildly wrong conclusion" if that conclusion is the same as yours (rejection of APDM)? What you mean is that he has another argument for the same conclusion. (And can you prove that your argument does not also presuppose an intuition about cardinalities?)"

Answer:

Mueller does more than rejecting APDM. He also suggests cardinal, interpersonal comparison of utility. The interpretation about "what I would mean", thus does not apply.

VTFD clarifies that the principle of "one man one vote" in itself implies a form of cardinal comparison, see p137, bottom. This observation is done within economic theory, without reference to Arrow's theorem.

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[9] A helpful comment would have been: "The conclusion on p251 might be read as if you might have a claim of an alternative proof for Arrow's Theorem, but you clearly do not intend this, since your discussion in 9.2.3 clearly has the additional presumption of majority rule (with some adaptation). Thus to prevent mis-reading of the conclusion, you might better say "an illustration of the proof of Arrow's Theorem that clearly illustrates the abuse by APDM"."

5.3. "The" solution

Student of deontic logic XYZ wrote (paraphrased): "If I agree with you on rejecting APDM then that does not mean that I share your conclusion that you have found "the" solution of Arrow's problem - after all, you still have to defend that the other conditions are all reasonable (and that is no sinecure: all conditions of Arrow have been criticized)."

Answer:

What I regard as "the" solution are the conclusions on VTFD:251, and not just dropping APDM.

I agree that all conditions have been discussed in the literature.

A new contribution on my part dating from 1990 is also to identify APDM as a confusion between voting and deciding. Vote counts satisfy APDM, as item *A* may have more vote counts than item *B*, independent from the vote counts on item *C*. Decisions need not satisfy APDM.

Conditions AU and AWP are a part of the definition of a SWF-GM.

VTFD is "*Voting theory for democracy*", and develops theory assuming AD (no dictator).

6. Closing

Presently there is no closing comment.

7. References

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