

Application of the Borda Fixed Point voting rule to the upcoming Dutch Parliamentary elections 2010

Thomas Colignatus, March 11 2010

On June 9 2010 the Dutch will have parliamentary elections. In March 2010 we see a discussion which party will get the greatest number of votes, with the assumption that this party will provide the Prime Minister to lead a coalition government. However, a Borda Fixed Point method would generate a different strategy to select the Prime Minister.

The Borda Fixed Point voting rule can be found in the book "Voting Theory for Democracy", <http://www.dataweb.nl/~cool/Papers/VTFD/Index.html>

See also <http://www.dataweb.nl/~cool/Papers/SocialWelfare/WithoutTimeNoMorality.html>

For Dutch readers there is also <http://www.dataweb.nl/~cool/SvHG/DOK/DOK-Aankondiging.html>

An analysis for 2006 selecting Rouvoet as Prime Minister instead of Balkenende can be found here: <http://www.dataweb.nl/~cool/Papers/SocialWelfare/BordaFP-DutchElections2006.pdf>

For 2010, with the currently available estimates, Rouvout again wins out, now with Rutte as second-in-line.

Start

```
Needs ["Economics `Pack`"]
```

```
ResetAll
```

```
Economics [Voting]
```

Data

Final results November 27 2006

```
Parties = {{CDA, 41}, {CU, 6}, {D66, 3}, {GL, 7}, {PvdA, 33}, {PvdD, 2}, {PVV, 9},  
           {SGP, 2}, {SP, 25}, {VVD, 22}}
```

```
( CDA 41 )  
 ( CU  6 )  
 ( D66 3 )  
 ( GL  7 )  
 ( PvdA 33 )  
 ( PvdD 2 )  
 ( PVV  9 )  
 ( SGP  2 )  
 ( SP  25 )  
 ( VVD 22 )
```

The following forecast can be taken from Maurice de Hond, March 6 2010, <http://maurice.oaip.nl/wp-content/uploads/2010/03/Landelijke-uitslag-Gemeenteraadsverkiezingen-2010.pdf>.

```

Parties = {{CDA, 25}, {CU, 7}, {D66, 18}, {GL, 13}, {PvdA, 24}, {PvdD, 2}, {PVV, 28},
  {SGP, 2}, {SP, 10}, {VVD, 19}, {TON, 2}} // Sort

( CDA 25
  CU 7
  D66 18
  GL 13
  PvdA 24
  PvdD 2
  PVV 28
  SGP 2
  SP 10
  TON 2
  VVD 19 )

Items = First /@ Parties
NumberOfItems = Length[Items]

{CDA, CU, D66, GL, PvdA, PvdD, PVV, SGP, SP, TON, VVD}

11

vlis = Last /@ Parties;
NumberOfVoters = Length[vlis];
Votes = vlis / Add[vlis]

{  $\frac{1}{6}$ ,  $\frac{7}{150}$ ,  $\frac{3}{25}$ ,  $\frac{13}{150}$ ,  $\frac{4}{25}$ ,  $\frac{1}{75}$ ,  $\frac{14}{75}$ ,  $\frac{1}{75}$ ,  $\frac{1}{15}$ ,  $\frac{1}{75}$ ,  $\frac{19}{150}$  }

% // NRoundAt[#, 2] &

{0.17, 0.05, 0.12, 0.09, 0.16, 0.01, 0.19, 0.01, 0.07, 0.01, 0.13}

StatusQuo []

CDA

```

Hypothesis

The statement of full preference orderings is a bit too complicated for the individual ballot box. However, the method can be used in Parliament by the 150 Members of Parliament.

The mathematical routines require party preferences on the selection of a Prime Minister (PM). Each party can present a candidate PM and then the Members of Parliament enter their orders of preference on the candidates. These preferences should best expressed not by the parties but by the individual Members of Parliament.

Parties might increase their chances by proposing candidates that are well received by other parties. It is simplest to presume that their candidates will be the leaders at the elections.

(NB. An alternative is to allow parties to present more candidates, proportional to the size of the party. A big party might present both its leader and some compromise candidates. However, since such compromise candidates would diminish the value of the leader, this is a less likely approach.)

Lacking those data we must enter an educated guess, and it is useful to assume some party homogeneity. We can maintain the same preference order as in 2006, adding TON at the bottom of all preferences except for PVV and TON itself.

```

Pref[CDA] = {CDA > CU > VVD > PvdD > GL > SP > SGP > PvdA > D66 > PVV > TON};
Pref[CU] = {CU > CDA > SGP > PvdA > GL > SP > VVD > PvdD > D66 > PVV > TON};
Pref[D66] = {D66 > PvdA > GL > VVD > PvdD > CU > SP > CDA > SGP > PVV > TON};
Pref[GL] = {GL > SP > PvdA > PvdD > D66 > CU > CDA > VVD > SGP > PVV > TON};
Pref[PvdA] = {PvdA > GL > D66 > SP > PvdD > CU > CDA > VVD > SGP > PVV > TON};
Pref[PvdD] = {PvdD > D66 > GL > CU > SP > PvdA > CDA > VVD > SGP > PVV > TON};
Pref[PVV] = {PVV > TON > VVD > CU > CDA > PvdD > SGP > SP > PvdA > D66 > GL};
Pref[SGP] = {SGP > CU > CDA > PvdD > VVD > PVV > SP > PvdA > GL > D66 > TON};
Pref[SP] = {SP > GL > PvdA > D66 > PvdD > CU > CDA > VVD > SGP > PVV > TON};
Pref[TON] = {TON > PVV > VVD > CU > CDA > PvdD > SGP > SP > PvdA > D66 > GL};
Pref[VVD] = {VVD > CDA > CU > D66 > PvdD > PVV > GL > PvdA > SP > SGP > TON};

```

These preference patterns can be translated in Borda ordinal preference scores.

```

Preferences = PrefToList[ToPref @@ Pref[#]] & /@ Items

```

$$\begin{pmatrix} 11 & 10 & 3 & 7 & 4 & 8 & 2 & 5 & 6 & 1 & 9 \\ 10 & 11 & 3 & 7 & 8 & 4 & 2 & 9 & 6 & 1 & 5 \\ 4 & 6 & 11 & 9 & 10 & 7 & 2 & 3 & 5 & 1 & 8 \\ 5 & 6 & 7 & 11 & 9 & 8 & 2 & 3 & 10 & 1 & 4 \\ 5 & 6 & 9 & 10 & 11 & 7 & 2 & 3 & 8 & 1 & 4 \\ 5 & 8 & 10 & 9 & 6 & 11 & 2 & 3 & 7 & 1 & 4 \\ 7 & 8 & 2 & 1 & 3 & 6 & 11 & 5 & 4 & 10 & 9 \\ 9 & 10 & 2 & 3 & 4 & 8 & 6 & 11 & 5 & 1 & 7 \\ 5 & 6 & 8 & 10 & 9 & 7 & 2 & 3 & 11 & 1 & 4 \\ 7 & 8 & 2 & 1 & 3 & 6 & 10 & 5 & 4 & 11 & 9 \\ 10 & 9 & 8 & 5 & 4 & 7 & 6 & 2 & 3 & 1 & 11 \end{pmatrix}$$

The Borda Fixed Point selection

Given the above data and assumptions the Borda Fixed Point algorithm determines the fixed point, i.e. the winner who also wins from the runner up (the alternative winner if the overall winner would not partake).

```

BordaFP[]

```

```

CU

```

```

BordaAnalysis[] // N

```

```

{Select → CU, BordaFPQ → {True},

```

```

WeightTotal → {7.2, 7.76, 6.11333, 6.78667, 6.62, 6.98, 4.34667, 3.99333, 6.1, 2.81333, 7.28667},

```

```

Position → ( 2. ), Ordering →

```

$$\left. \begin{array}{l} 2.81333 \text{ TON} \\ 3.99333 \text{ SGP} \\ 4.34667 \text{ PVV} \\ 6.1 \text{ SP} \\ 6.11333 \text{ D66} \\ 6.62 \text{ PvdA} \\ 6.78667 \text{ GL} \\ 6.98 \text{ PvdD} \\ 7.2 \text{ CDA} \\ 7.28667 \text{ VVD} \\ 7.76 \text{ CU} \end{array} \right\}$$

CU (Rouvoet) would not only have most votes in a Borda vote but would also win in a (binary) duel from VVD (Rutte), where the VVD would be selected if CU would not partake.

Alternative: Pairwise voting

It appears that the CU is also the Condorcet winner - i.e. wins from all pairwise votes.

This criterion however is not a strong one since there can be elections where there is no such winner or there can be elections where that winner loses in a Borda approach.

PairwiseMajority []

$$\left\{ \text{VoteMargin} \rightarrow \text{VoteMargin} \right\} \begin{pmatrix} 0 & -\frac{31}{75} & \frac{8}{75} & \frac{8}{75} & \frac{8}{75} & \frac{8}{75} & \frac{3}{5} & \frac{73}{75} & \frac{8}{75} & \frac{3}{5} & \frac{8}{75} \\ \frac{31}{75} & 0 & \frac{8}{75} & \frac{8}{75} & \frac{2}{15} & \frac{8}{75} & \frac{3}{5} & \frac{73}{75} & \frac{28}{75} & \frac{3}{5} & \frac{8}{75} \\ -\frac{8}{75} & -\frac{8}{75} & 0 & -\frac{2}{25} & -\frac{12}{25} & -\frac{4}{75} & \frac{43}{75} & \frac{11}{75} & -\frac{4}{25} & \frac{3}{5} & -\frac{8}{75} \\ -\frac{8}{75} & -\frac{8}{75} & \frac{2}{25} & 0 & -\frac{2}{25} & -\frac{1}{25} & \frac{8}{25} & \frac{12}{25} & \frac{11}{25} & \frac{3}{5} & -\frac{1}{75} \\ -\frac{8}{75} & -\frac{2}{15} & \frac{12}{25} & \frac{2}{25} & 0 & -\frac{1}{25} & \frac{8}{25} & \frac{11}{75} & -\frac{7}{75} & \frac{3}{5} & -\frac{1}{75} \\ -\frac{8}{75} & -\frac{8}{75} & \frac{4}{75} & \frac{1}{25} & \frac{1}{25} & 0 & \frac{3}{5} & \frac{22}{25} & \frac{7}{25} & \frac{3}{5} & -\frac{8}{25} \\ -\frac{3}{5} & -\frac{3}{5} & -\frac{43}{75} & -\frac{8}{25} & -\frac{8}{25} & -\frac{3}{5} & 0 & -\frac{26}{75} & -\frac{8}{25} & \frac{73}{75} & -\frac{3}{5} \\ -\frac{73}{75} & \frac{73}{75} & -\frac{11}{75} & -\frac{12}{25} & -\frac{11}{75} & -\frac{22}{25} & \frac{26}{75} & 0 & -\frac{12}{25} & \frac{3}{5} & -\frac{22}{25} \\ -\frac{8}{75} & \frac{28}{75} & \frac{4}{25} & -\frac{11}{25} & \frac{7}{75} & -\frac{7}{25} & \frac{8}{25} & \frac{12}{25} & 0 & \frac{3}{5} & -\frac{19}{75} \\ -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{73}{75} & -\frac{3}{5} & -\frac{3}{5} & 0 & -\frac{3}{5} \\ -\frac{8}{75} & -\frac{8}{75} & \frac{8}{75} & \frac{1}{75} & \frac{1}{75} & \frac{8}{25} & \frac{3}{5} & \frac{22}{25} & \frac{19}{75} & \frac{3}{5} & 0 \end{pmatrix},$$

1 → {StatusQuo → CDA, Sum → {9, 10, 3, 5, 5, 7, 1, 2, 5, 0, 8}, Max → 10, Condorcet winner → CU,
 Pref → Pref(TON, PVV, SGP, D66, {GL, PvdA, SP}, PvdD, VVD, CDA, CU), Find → CU,
 LastCycleTest → False, Select → CU}, N → {Sum → { $\frac{12}{5}, \frac{88}{25}, \frac{17}{75}, \frac{118}{75}, \frac{31}{25}, \frac{49}{25}, -\frac{248}{75}, -\frac{301}{75}, \frac{1}{5}, -\frac{478}{75}, \frac{193}{75}$ },
 Pref → Pref(TON, SGP, PVV, SP, D66, PvdA, GL, PvdD, CDA, VVD, CU), Select → CU}, All → CU}

Alternative: Plurality voting

Plurality selects the person with the highest vote - that might be less than 50%. All parties vote for their own candidate and there is no clear winner.

Plurality []

$$\left\{ \text{Sum} \rightarrow \begin{pmatrix} \text{CDA} & \frac{1}{6} \\ \text{CU} & \frac{7}{150} \\ \text{D66} & \frac{3}{25} \\ \text{GL} & \frac{13}{150} \\ \text{PvdA} & \frac{4}{25} \\ \text{PvdD} & \frac{1}{75} \\ \text{PVV} & \frac{14}{75} \\ \text{SGP} & \frac{1}{75} \\ \text{SP} & \frac{1}{15} \\ \text{TON} & \frac{1}{75} \\ \text{VVD} & \frac{19}{150} \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} \frac{1}{75} & \text{PvdD} \\ \frac{1}{75} & \text{SGP} \\ \frac{1}{75} & \text{TON} \\ \frac{7}{150} & \text{CU} \\ \frac{1}{15} & \text{SP} \\ \frac{13}{150} & \text{GL} \\ \frac{3}{25} & \text{D66} \\ \frac{19}{150} & \text{VVD} \\ \frac{4}{25} & \text{PvdA} \\ \frac{1}{6} & \text{CDA} \\ \frac{14}{75} & \text{PVV} \end{pmatrix}, \text{Max} \rightarrow \left\{ \text{PVV}, \frac{14}{75} \right\}, \text{Select} \rightarrow \{ \} \right\}$$

$$\% // N$$

$$\left\{ \text{Sum} \rightarrow \begin{pmatrix} \text{CDA} & 0.166667 \\ \text{CU} & 0.0466667 \\ \text{D66} & 0.12 \\ \text{GL} & 0.0866667 \\ \text{PvdA} & 0.16 \\ \text{PvdD} & 0.0133333 \\ \text{PVV} & 0.186667 \\ \text{SGP} & 0.0133333 \\ \text{SP} & 0.0666667 \\ \text{TON} & 0.0133333 \\ \text{VVD} & 0.126667 \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} 0.0133333 & \text{PvdD} \\ 0.0133333 & \text{SGP} \\ 0.0133333 & \text{TON} \\ 0.0466667 & \text{CU} \\ 0.0666667 & \text{SP} \\ 0.0866667 & \text{GL} \\ 0.12 & \text{D66} \\ 0.126667 & \text{VVD} \\ 0.16 & \text{PvdA} \\ 0.166667 & \text{CDA} \\ 0.186667 & \text{PVV} \end{pmatrix}, \text{Max} \rightarrow \{\text{PVV}, 0.186667\}, \text{Select} \rightarrow \{\} \right\}$$

An example pairwise vote

The following example shows that the candidate of the CU would win from the candidate of the CDA in a pairwise vote.

There are however 55 of such pairwise votes and thus it is simplest if all Members of Parliament would enter a single preference list (as shown above) whereafter the algorithm determines the overall result.

SelectPreferences [{CDA, CU}]

CheckVote::adj : NumberOfItems adjusted to 2

{Number of Voters → 11, Number of items → 2, Votes are nonnegative and add up to 1 → True,
 Preferences fit the numbers of Voters and Items → True, Type of scale → Ordinal, Preferences give a proper ordering → True,
 Preferences add up to → {3}, Items → {CDA, CU}, Votes → $\left\{ \frac{1}{6}, \frac{7}{150}, \frac{3}{25}, \frac{13}{150}, \frac{4}{25}, \frac{1}{75}, \frac{14}{75}, \frac{1}{75}, \frac{1}{15}, \frac{1}{75}, \frac{19}{150} \right\}$ }

Plurality []

{Sum → $\begin{pmatrix} \text{CDA} & \frac{22}{75} \\ \text{CU} & \frac{53}{75} \end{pmatrix}$, Ordering → $\begin{pmatrix} \frac{22}{75} & \text{CDA} \\ \frac{53}{75} & \text{CU} \end{pmatrix}$, Max → $\left\{ \text{CU}, \frac{53}{75} \right\}$, Select → CU}

Conclusion

The simplest scheme is where parties vote for their own candidate. Then the PVV will get the highest score, which is still only 19% of the vote. Thus, "simplest" doesn't seem to be too useful.

In pairwise voting it so happens that the CU is the Condorcet winner.

However, that kind of voting is notoriously unstable. In many elections there is no such winner, leaving one with the question what to do next.

The overall best approach very likely is the Borda Fixed Point. In this case this coincides with the Condorcet winner since the CU apparently is rather high on the preference lists anyway.

Of course, voting would be conditional on agreements on policy and coalition forming. However, in "Voting Theory for Democracy" it appears that a Cabinet "mirroring" Parliament would tend to be best, which also means that the issue on policy making could be rather distinct from the selection of the Prime Minister.

Appendix: Strategic voting

Strategic voting can never be fully avoided.

VVD might give its competitor CU much less weight and then itself becomes the Borda Fixed Point.

```
Pref[VVD] = {VVD > CDA > D66 > PvdD > PVV > GL > PvdA > SP > SGP > TON > CU};
```

```
Preferences = PrefToList[ToPref @@ Pref[#]] & /@ Items
```

$$\begin{pmatrix} 11 & 10 & 3 & 7 & 4 & 8 & 2 & 5 & 6 & 1 & 9 \\ 10 & 11 & 3 & 7 & 8 & 4 & 2 & 9 & 6 & 1 & 5 \\ 4 & 6 & 11 & 9 & 10 & 7 & 2 & 3 & 5 & 1 & 8 \\ 5 & 6 & 7 & 11 & 9 & 8 & 2 & 3 & 10 & 1 & 4 \\ 5 & 6 & 9 & 10 & 11 & 7 & 2 & 3 & 8 & 1 & 4 \\ 5 & 8 & 10 & 9 & 6 & 11 & 2 & 3 & 7 & 1 & 4 \\ 7 & 8 & 2 & 1 & 3 & 6 & 11 & 5 & 4 & 10 & 9 \\ 9 & 10 & 2 & 3 & 4 & 8 & 6 & 11 & 5 & 1 & 7 \\ 5 & 6 & 8 & 10 & 9 & 7 & 2 & 3 & 11 & 1 & 4 \\ 7 & 8 & 2 & 1 & 3 & 6 & 10 & 5 & 4 & 11 & 9 \\ 10 & 1 & 9 & 6 & 5 & 8 & 7 & 3 & 4 & 2 & 11 \end{pmatrix}$$

```
BordaFP[]
```

```
VVD
```

```
BordaAnalysis[] // N
```

```
{Select → VVD, BordaFPQ → {True},
```

```
WeightTotal → {7.2, 6.74667, 6.24, 6.91333, 6.74667, 7.10667, 4.47333, 4.12, 6.22667, 2.94, 7.28667},
```

```
Position → ( 11. ), Ordering →
```

$$\left. \begin{array}{l} 2.94 \quad \text{TON} \\ 4.12 \quad \text{SGP} \\ 4.47333 \quad \text{PVV} \\ 6.22667 \quad \text{SP} \\ 6.24 \quad \text{D66} \\ 6.74667 \quad \text{CU} \\ 6.74667 \quad \text{PvdA} \\ 6.91333 \quad \text{GL} \\ 7.10667 \quad \text{PvdD} \\ 7.2 \quad \text{CDA} \\ 7.28667 \quad \text{VVD} \end{array} \right\}$$

However, other parties might anticipate such VVD strategic voting behaviour and they might respond by entering CU much higher in their preferences.

```
Pref[CDA] = {CDA > CU > VVD > PvdD > GL > SP > SGP > PvdA > D66 > PVV > TON};
```

```
Pref[CU] = {CU > CDA > SGP > PvdA > GL > SP > VVD > PvdD > D66 > PVV > TON};
```

```
Pref[D66] = {D66 > CU > PvdA > GL > VVD > PvdD > SP > CDA > SGP > PVV > TON};
```

```
Pref[GL] = {GL > CU > SP > PvdA > PvdD > D66 > CDA > VVD > SGP > PVV > TON};
```

```
Pref[PvdA] = {PvdA > CU > GL > D66 > SP > PvdD > CDA > VVD > SGP > PVV > TON};
```

```
Pref[PvdD] = {PvdD > CU > D66 > GL > SP > PvdA > CDA > VVD > SGP > PVV > TON};
```

```
Pref[PVV] = {PVV > CU > TON > VVD > CDA > PvdD > SGP > SP > PvdA > D66 > GL};
```

```
Pref[SGP] = {SGP > CU > CDA > PvdD > VVD > PVV > SP > PvdA > GL > D66 > TON};
```

```
Pref[SP] = {SP > CU > GL > PvdA > D66 > PvdD > CDA > VVD > SGP > PVV > TON};
```

```
Pref[TON] = {TON > CU > PVV > VVD > CDA > PvdD > SGP > SP > PvdA > D66 > GL};
```

```
Pref[VVD] = {VVD > CDA > D66 > PvdD > PVV > GL > PvdA > SP > SGP > TON > CU};
```

```
Preferences = PrefToList[ToPref @@ Pref[#]] & /@ Items
```

```
( 11 10 3 7 4 8 2 5 6 1 9 )
( 10 11 3 7 8 4 2 9 6 1 5 )
( 4 10 11 8 9 6 2 3 5 1 7 )
( 5 10 6 11 8 7 2 3 9 1 4 )
( 5 10 8 9 11 6 2 3 7 1 4 )
( 5 10 9 8 6 11 2 3 7 1 4 )
( 7 10 2 1 3 6 11 5 4 9 8 )
( 9 10 2 3 4 8 6 11 5 1 7 )
( 5 10 7 9 8 6 2 3 11 1 4 )
( 7 10 2 1 3 6 9 5 4 11 8 )
( 10 1 9 6 5 8 7 3 4 2 11 )
```

```
BordaFP[]
```

```
CU
```

```
BordaAnalysis[] // N
```

```
{Select → CU, BordaFPQ → {True},
```

```
WeightTotal → {7.2, 8.90667, 5.91333, 6.55333, 6.47333, 6.67333, 4.46, 4.12, 5.98, 2.75333, 6.96667},
```

```
Position → ( 2. ), Ordering → ( 2.75333 TON )
( 4.12 SGP )
( 4.46 PVV )
( 5.91333 D66 )
( 5.98 SP )
( 6.47333 PvdA )
( 6.55333 GL )
( 6.67333 PvdD )
( 6.96667 VVD )
( 7.2 CDA )
( 8.90667 CU )
```

A way to reduce strategic voting is to publish the votes, so that parties may have some explaining to do. A secret ballot would hold for the individual voter in the ballot box but not necessarily for voting by Members of Parliament on the Prime Minister. Such open statements of preference do not exclude strategic voting but they do somewhat reduce it. The element of strategy would be reduced even more when preference orderings are announced before the national elections so that there is less room for tinkering after the elections.

Overall, the political discussion and the selection of the Prime Minister of the coalition cabinet would seem more sophisticated when using orderings and Borda Fixed Point than merely taking the leader of the largest party. It would also be advisable to have the cabinet mirror the distribution in Parliament, since one would need a good argument to exclude a party with say 5% of the votes from partaking in government.