

# Application of the Borda Fixed Point voting rule to the Dutch Parliamentary elections 2006

Thomas Colignatus November 23 & 28 2006

The Borda Fixed Point voting rule can be found in the book "Voting Theory for Democracy",  
<http://www.dataweb.nl/~cool/Papers/VTFD/Index.html>

See also <http://www.dataweb.nl/~cool/Papers/SocialWelfare/WithoutTimeNoMorality.html>

For Dutch readers there is also <http://www.dataweb.nl/~cool/SvHG/DOK/DOK-Aankondiging.html>

---

## Start

```
Needs ["Economics`Pack`"]
```

```
ResetAll
```

```
Economics [Voting]
```

---

## Data

Final results November 27 2006

```
Parties = {{CDA, 41}, {CU, 6}, {D66, 3}, {GL, 7}, {PvdA, 33}, {PvdD, 2}, {PvdV, 9},
           {SGP, 2}, {SP, 25}, {VVD, 22}}
```

```
(CDA 41)
(CU 6)
(D66 3)
(GL 7)
(PvdA 33)
(PvdD 2)
(PvdV 9)
(SGP 2)
(SP 25)
(VVD 22)
```

```
Items = First /@ Parties
```

```
NumberOfItems = Length[Items]
```

```
{CDA, CU, D66, GL, PvdA, PvdD, PvdV, SGP, SP, VVD}
```

```

vlis = Last /@ Parties;
NumberOfVoters = Length[vlis];
Votes = vlis / Add[vlis]

{ 41/150, 1/25, 1/50, 7/150, 11/50, 1/75, 3/50, 1/75, 1/6, 11/75 }

StatusQuo[]

CDA

```

---

## Hypothesis

These routines require party preferences on the selection of a Prime Minister. Each party can present a candidate PM and then the Members of Parliament enter their orders of preference on the candidates. These preferences should best expressed not by the parties but by the individual Members of Parliament.

Parties might increase their chances by proposing candidates that are well received by other parties. Perhaps it is simplest though to presume that their candidates will be the leaders at the elections.

Lacking those data we must enter an educated guess, and it is useful to assume some party homogeneity.

```

Pref[CDA] = {CDA > CU > VVD > PvdD > GL > SP > SGP > PvdA > D66 > PvdV};
Pref[CU] = {CU > CDA > SGP > PvdA > GL > SP > VVD > PvdD > D66 > PvdV};
Pref[D66] = {D66 > PvdA > GL > VVD > PvdD > CU > SP > CDA > SGP > PvdV};
Pref[GL] = {GL > SP > PvdA > PvdD > D66 > CU > CDA > VVD > SGP > PvdV};
Pref[PvdA] = {PvdA > GL > D66 > SP > PvdD > CU > CDA > VVD > SGP > PvdV};
Pref[PvdD] = {PvdD > D66 > GL > CU > SP > PvdA > CDA > VVD > SGP > PvdV};
Pref[PvdV] = {PvdV > VVD > CU > CDA > PvdD > SGP > SP > PvdA > D66 > GL};
Pref[SGP] = {SGP > CU > CDA > PvdD > VVD > PvdV > SP > PvdA > GL > D66};
Pref[SP] = {SP > GL > PvdA > D66 > PvdD > CU > CDA > VVD > SGP > PvdV};
Pref[VVD] = {VVD > CDA > CU > D66 > PvdD > PvdV > GL > PvdA > SP > SGP};

```

These preference patterns can be translated in Borda ordinal preference scores.

```

Preferences = PrefToList[ToPref @@ Pref[#]] & /@ Items

```

$$\begin{pmatrix} 10 & 9 & 2 & 6 & 3 & 7 & 1 & 4 & 5 & 8 \\ 9 & 10 & 2 & 6 & 7 & 3 & 1 & 8 & 5 & 4 \\ 3 & 5 & 10 & 8 & 9 & 6 & 1 & 2 & 4 & 7 \\ 4 & 5 & 6 & 10 & 8 & 7 & 1 & 2 & 9 & 3 \\ 4 & 5 & 8 & 9 & 10 & 6 & 1 & 2 & 7 & 3 \\ 4 & 7 & 9 & 8 & 5 & 10 & 1 & 2 & 6 & 3 \\ 7 & 8 & 2 & 1 & 3 & 6 & 10 & 5 & 4 & 9 \\ 8 & 9 & 1 & 2 & 3 & 7 & 5 & 10 & 4 & 6 \\ 4 & 5 & 7 & 9 & 8 & 6 & 1 & 2 & 10 & 3 \\ 9 & 8 & 7 & 4 & 3 & 6 & 5 & 1 & 2 & 10 \end{pmatrix}$$

---

## The Borda Fixed Point selection

Given the above data and assumptions the Borda Fixed Point algorithm determines that fixed point, i.e. the winner who also wins from the runner up (the alternative winner if the overall winner would not partake).

```
BordaFP [ ]
```

```
CU
```

```
BordaAnalysis [ ] // N
```

```
{Select → CU, BordaFPQ → {True},
```

```
WeightTotal → {6.78667, 6.99333, 5.31333, 6.76667, 5.91333, 6.26667, 2.18, 2.92667, 5.94, 5.91333},
```

```
Position → (2.), Ordering →
```

2.18	PvdV
2.92667	SGP
5.31333	D66
5.91333	PvdA
5.91333	VVD
5.94	SP
6.26667	PvdD
6.76667	GL
6.78667	CDA
6.99333	CU

---

## Alternative: Pairwise voting

It appears that the CU is also the Condorcet winner - i.e. wins from all pairwise votes.

This criterion however is not a strong one since there can be elections where there is no such winner or there can be elections where that winner loses in a Borda approach.

**PairwiseMajority[]**

VoteMarginToPref:cyc : Cycle {PvdD, D66, GL, PvdD}

$$\left\{ \text{VoteMargin} \rightarrow \text{VoteMargin} \right\} \left( \begin{array}{cccccccccc} 0 & -\frac{4}{25} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{22}{25} & \frac{73}{75} & \frac{1}{15} & \frac{41}{75} \\ \frac{4}{25} & 0 & \frac{1}{15} & \frac{1}{15} & \frac{7}{75} & \frac{1}{15} & \frac{22}{25} & \frac{73}{75} & \frac{2}{15} & \frac{41}{75} \\ -\frac{1}{15} & -\frac{1}{15} & 0 & -\frac{13}{25} & -\frac{16}{25} & \frac{8}{75} & \frac{64}{75} & \frac{17}{75} & -\frac{1}{5} & -\frac{1}{15} \\ -\frac{1}{15} & -\frac{1}{15} & \frac{13}{25} & 0 & \frac{22}{75} & -\frac{1}{75} & \frac{14}{25} & \frac{58}{75} & \frac{13}{25} & \frac{1}{75} \\ -\frac{1}{15} & -\frac{7}{75} & \frac{16}{25} & -\frac{22}{75} & 0 & -\frac{1}{75} & \frac{14}{25} & \frac{17}{75} & -\frac{11}{75} & \frac{1}{75} \\ -\frac{1}{15} & -\frac{1}{15} & -\frac{8}{75} & \frac{1}{75} & \frac{1}{75} & 0 & \frac{22}{25} & \frac{67}{75} & \frac{4}{75} & -\frac{2}{25} \\ -\frac{22}{25} & -\frac{22}{25} & -\frac{64}{75} & -\frac{14}{25} & -\frac{14}{25} & -\frac{22}{25} & 0 & -\frac{44}{75} & -\frac{14}{25} & -\frac{22}{25} \\ -\frac{73}{75} & -\frac{73}{75} & -\frac{17}{75} & -\frac{58}{75} & -\frac{17}{75} & -\frac{67}{75} & \frac{44}{75} & 0 & -\frac{58}{75} & -\frac{67}{75} \\ -\frac{1}{15} & -\frac{2}{15} & \frac{1}{5} & -\frac{13}{25} & \frac{11}{75} & -\frac{4}{75} & \frac{14}{25} & \frac{58}{75} & 0 & -\frac{2}{75} \\ -\frac{41}{75} & -\frac{41}{75} & \frac{1}{15} & -\frac{1}{75} & -\frac{1}{75} & \frac{2}{25} & \frac{22}{25} & \frac{67}{75} & \frac{2}{75} & 0 \end{array} \right)$$

1 → {StatusQuo → CDA, Sum → {8, 9, 3, 6, 4, 5, 0, 1, 4, 5}, Max → 9, Condorcet winner → CU,  
 Pref → Pref(PvdV, SGP, {D66, GL, PvdA, PvdD, SP, VVD}, CDA, CU), Find → CU, LastCycleTest → False,  
 Select → CU}, N → {Sum → { $\frac{193}{75}$ ,  $\frac{224}{75}$ ,  $-\frac{28}{75}$ ,  $\frac{38}{15}$ ,  $\frac{62}{75}$ ,  $\frac{23}{15}$ ,  $-\frac{166}{25}$ ,  $-\frac{386}{75}$ ,  $\frac{22}{25}$ ,  $\frac{62}{75}$ },  
 Pref → Pref(PvdV, SGP, D66, {PvdA, VVD}, SP, PvdD, GL, CDA, CU), Select → CU}, All → CU}

**Alternative: Plurality voting**

Plurality selects the person with the highest vote - that might be less than 50%. All parties vote for their own candidate and there is no clear winner.

**Plurality[]**

$$\left\{ \text{Sum} \rightarrow \left( \begin{array}{l} \text{CDA} \quad \frac{41}{150} \\ \text{CU} \quad \frac{1}{25} \\ \text{D66} \quad \frac{1}{50} \\ \text{GL} \quad \frac{7}{150} \\ \text{PvdA} \quad \frac{11}{50} \\ \text{PvdD} \quad \frac{1}{75} \\ \text{PvdV} \quad \frac{3}{50} \\ \text{SGP} \quad \frac{1}{75} \\ \text{SP} \quad \frac{1}{6} \\ \text{VVD} \quad \frac{11}{75} \end{array} \right), \text{Ordering} \rightarrow \left( \begin{array}{l} \frac{1}{75} \quad \text{PvdD} \\ \frac{1}{75} \quad \text{SGP} \\ \frac{1}{50} \quad \text{D66} \\ \frac{1}{25} \quad \text{CU} \\ \frac{7}{150} \quad \text{GL} \\ \frac{3}{50} \quad \text{PvdV} \\ \frac{11}{75} \quad \text{VVD} \\ \frac{1}{6} \quad \text{SP} \\ \frac{11}{50} \quad \text{PvdA} \\ \frac{41}{150} \quad \text{CDA} \end{array} \right), \text{Max} \rightarrow \left\{ \text{CDA}, \frac{41}{150} \right\}, \text{Select} \rightarrow \{ \} \right\}$$

% // N

$$\left\{ \text{Sum} \rightarrow \begin{pmatrix} \text{CDA} & 0.273333 \\ \text{CU} & 0.04 \\ \text{D66} & 0.02 \\ \text{GL} & 0.0466667 \\ \text{PvdA} & 0.22 \\ \text{PvdD} & 0.0133333 \\ \text{PvdV} & 0.06 \\ \text{SGP} & 0.0133333 \\ \text{SP} & 0.166667 \\ \text{VVD} & 0.146667 \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} 0.0133333 & \text{PvdD} \\ 0.0133333 & \text{SGP} \\ 0.02 & \text{D66} \\ 0.04 & \text{CU} \\ 0.0466667 & \text{GL} \\ 0.06 & \text{PvdV} \\ 0.146667 & \text{VVD} \\ 0.166667 & \text{SP} \\ 0.22 & \text{PvdA} \\ 0.273333 & \text{CDA} \end{pmatrix}, \text{Max} \rightarrow \{\text{CDA}, 0.273333\}, \text{Select} \rightarrow \{\} \right\}$$

## An example pairwise vote

The following example shows that the candidate of the CU would win from the candidate of the CDA in a pairwise vote.

There are however 45 of such pairwise votes and thus it is simplest if all Members of Parliament would enter a single preference list whereafter the algorithm determines the overall result.

**SelectPreferences**[{CDA, CU}]

CheckVote::adj : NumberOfItems adjusted to 2

{Number of Voters → 10, Number of items → 2, Votes are nonnegative and add up to 1 → True,  
 Preferences fit the numbers of Voters and Items → True, Type of scale → Ordinal,  
 Preferences give a proper ordering → True, Preferences add up to → {3},  
 Items → {CDA, CU}, Votes →  $\left\{ \frac{41}{150}, \frac{1}{25}, \frac{1}{50}, \frac{7}{150}, \frac{11}{50}, \frac{1}{75}, \frac{3}{50}, \frac{1}{75}, \frac{1}{6}, \frac{11}{75} \right\}$ }

**Plurality**[ ]

{Sum →  $\begin{pmatrix} \text{CDA} & \frac{21}{50} \\ \text{CU} & \frac{29}{50} \end{pmatrix}$ , Ordering →  $\begin{pmatrix} \frac{21}{50} & \text{CDA} \\ \frac{29}{50} & \text{CU} \end{pmatrix}$ , Max →  $\left\{ \text{CU}, \frac{29}{50} \right\}$ , Select → CU}

## Conclusion

The simplest scheme is where parties vote for their own candidate. Then the CDA will get the highest score, which is still only 27.3% of the vote. Thus, "simplest" doesn't seem to be too useful.

In pairwise voting it so happens that the CU is the Condorcet winner.

However, that kind of voting is notoriously unstable. In many elections there is no such winner, leaving one with the question what to do next.

The overall best approach is the Borda Fixed Point. In this case this coincides with the Condorcet winner since the CU apparently is rather high on the preference lists anyway.

Of course, voting would be conditional on agreements on policy and coalition forming. However, in "Voting Theory for Democracy" it appears that a Cabinet "mirroring" Parliament would tend to be best, so that the issue on policy making could still be rather distinct from the selection of the Prime Minister.

## Appendix: Strategic voting

Strategic voting can never be fully avoided.

CDA might give its competitor CU much less weight and then itself becomes the Borda Fixed Point.

```
Pref[CDA] = {CDA > VVD > PvdD > GL > SP > SGP > PvdA > D66 > PvdV > CU};
```

```
Preferences = PrefToList[ToPref @@ Pref[#]] & /@ Items
```

$$\begin{pmatrix} 10 & 1 & 3 & 7 & 4 & 8 & 2 & 5 & 6 & 9 \\ 9 & 10 & 2 & 6 & 7 & 3 & 1 & 8 & 5 & 4 \\ 3 & 5 & 10 & 8 & 9 & 6 & 1 & 2 & 4 & 7 \\ 4 & 5 & 6 & 10 & 8 & 7 & 1 & 2 & 9 & 3 \\ 4 & 5 & 8 & 9 & 10 & 6 & 1 & 2 & 7 & 3 \\ 4 & 7 & 9 & 8 & 5 & 10 & 1 & 2 & 6 & 3 \\ 7 & 8 & 2 & 1 & 3 & 6 & 10 & 5 & 4 & 9 \\ 8 & 9 & 1 & 2 & 3 & 7 & 5 & 10 & 4 & 6 \\ 4 & 5 & 7 & 9 & 8 & 6 & 1 & 2 & 10 & 3 \\ 9 & 8 & 7 & 4 & 3 & 6 & 5 & 1 & 2 & 10 \end{pmatrix}$$

```
BordaFP[]
```

```
BordaFP::chg : Borda gave {GL}, the selected Fixed Point is {CDA}
```

```
CDA
```

```
BordaAnalysis[] // N
```

```
{Select → GL, BordaFPQ → {False},
```

```
WeightTotal → {6.78667, 4.80667, 5.58667, 7.04, 6.18667, 6.54, 2.45333, 3.2, 6.21333, 6.18667},
```

```
Position → (4.), Ordering →
```

$$\left. \begin{array}{l} 2.45333 \text{ PvdV} \\ 3.2 \text{ SGP} \\ 4.80667 \text{ CU} \\ 5.58667 \text{ D66} \\ 6.18667 \text{ PvdA} \\ 6.18667 \text{ VVD} \\ 6.21333 \text{ SP} \\ 6.54 \text{ PvdD} \\ 6.78667 \text{ CDA} \\ 7.04 \text{ GL} \end{array} \right\}$$

However, as other parties might anticipate such CDA strategic voting behaviour they might respond by entering CU much higher in their preferences.

```

Pref[CDA] = {CDA > VVD > PvdD > GL > SP > SGP > PvdA > D66 > PvdV > CU};
Pref[CU] = {CU > CDA > SGP > PvdA > GL > SP > VVD > PvdD > D66 > PvdV};
Pref[D66] = {D66 > CU > PvdA > GL > VVD > PvdD > SP > CDA > SGP > PvdV};
Pref[GL] = {GL > CU > SP > PvdA > PvdD > D66 > CDA > VVD > SGP > PvdV};
Pref[PvdA] = {PvdA > CU > GL > D66 > SP > PvdD > CDA > VVD > SGP > PvdV};
Pref[PvdD] = {PvdD > CU > D66 > GL > SP > PvdA > CDA > VVD > SGP > PvdV};
Pref[PvdV] = {PvdV > CU > VVD > CDA > PvdD > SGP > SP > PvdA > D66 > GL};
Pref[SGP] = {SGP > CU > CDA > PvdD > VVD > PvdV > SP > PvdA > GL > D66};
Pref[SP] = {SP > CU > GL > PvdA > D66 > PvdD > CDA > VVD > SGP > PvdV};
Pref[VVD] = {VVD > CU > CDA > D66 > PvdD > PvdV > GL > PvdA > SP > SGP};

```

```

Preferences = PrefToList[ToPref @@ Pref[#]] & /@ Items

```

```

( 10 1 3 7 4 8 2 5 6 9
  9 10 2 6 7 3 1 8 5 4
  3 9 10 7 8 5 1 2 4 6
  4 9 5 10 7 6 1 2 8 3
  4 9 7 8 10 5 1 2 6 3
  4 9 8 7 5 10 1 2 6 3
  7 9 2 1 3 6 10 5 4 8
  8 9 1 2 3 7 5 10 4 6
  4 9 6 8 7 5 1 2 10 3
  8 9 7 4 3 6 5 1 2 10 )

```

```

BordaFP[]

```

```

BordaFP::set : Local set found: {CDA, CU}

```

```

BordaFP::chg : Borda gave {CU}, the selected Fixed Point is CU

```

```

CU

```

```

BordaAnalysis[] // N

```

```

{Select → CU, BordaFPQ → {True},

```

```

  WeightTotal → {6.64, 6.85333, 5.14, 6.62, 5.95333, 6.08667, 2.45333, 3.2, 5.94667, 6.10667},

```

```

  Position → (2.), Ordering →
  ( 2.45333 PvdV
    3.2 SGP
    5.14 D66
    5.94667 SP
    5.95333 PvdA
    6.08667 PvdD
    6.10667 VVD
    6.62 GL
    6.64 CDA
    6.85333 CU )

```