Single vote multiple seats elections. Supplement for the Dutch elections of June 9 2010

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http://www.dataweb.nl/~cool

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1. Introduction

The discussion can be found in Colignatus (2010), “Single vote multiple seats elections. Didactics of district versus proportional representation, using the examples of the United Kingdom and The Netherlands”, http://mpra.ub.uni-muenchen.de/22782/.

This present notebook only supplements for the Dutch June 9 2010 elections. The 2006 results already included a ruling coalition but at the moment of writing the negotiations for 2010 are still in progress.

The number of voters that a party gains can be divided by the electoral quota to generate the number of seats. Generally there will be a remainder of voters, not sufficient to generate a full seat. These remainders for all parties either cause empty seats or can be reallocated by an algorithm. The crucial question is how to interprete such algorithm.

2. Dutch Parliamentary elections in 2010

■ The official results

In the June 9 2010 Dutch Parliamentary elections 12.5 million registered voters chose 150 Members of Parliament. The electoral quota is 1 / 150 = 0.67% of the electoral vote, though taken from the turnout of 75% (valid votes). A total of 2 seats was wasted on invalid or blanco votes and parties that did not pass the threshold of the electoral quota. To reduce idiosyncracy the scores can be expressed per 100,000 but we use the true scores since we also intend to show that the routines can approximate the true national result.

```
MultipleSeatsCase[Set, 312 010];
lis = MultipleSeatsCase[];
TableForm[lis, TableAlignments → Right]
```

```
                 Dutch Parliament 2010      Voters  Per 100000  Percentage  Seats  Label
     Christen Democratisch Appèl (CDA)   1 281 886      10 235       13.61     21      A
           Partij van de Arbeid (PvdA)   1 848 805      14 762       19.63     30      B
            SP (Socialistische Partij)     924 696        7383        9.82     15      C
                                   VVD   1 929 575      15 407       20.49     31      D
         PVV (Partij voor de Vrijheid)   1 454 493      11 614       15.45     24      E
                            Groenlinks     628 096        5015        6.67     10      F
                     ChristenUnie (CU)     305 094        2436        3.24      5      G
                   Democraten 66 (D66)     654 167        5223        6.95     10      H
                 Partij voor de Dieren     122 317         977         1.3      2      I
Staatkundig Gereformeerde Partij (SGP)     163 581        1306        1.74      2      J
                       Nieuw Nederland        2010          16        0.02      0      K
Trots op Nederland. Lijst Rita Verdonk      52 937         423        0.56      0      L
     Partij voor Mens en Spirit (MenS)      26 196         209        0.28      0      M
                               Heel NL        1255          10        0.01      0      N
                            Partij een        2042          16        0.02      0      O
                              Lijst 17        7456          60        0.08      0      P
                         Piratenpartij      10 471          84        0.11      0      Q
                              Lijst 19         924           7        0.01      0      R
                           Valid votes   9 416 001      75 183         100    150      α
             PM. Below electoral quota     103 291         825         1.1    1.6      β
   PM. Wasted (invalid or below quota)     130 267        1040        1.38    2.1   Void
          Invalid votes (incl. blanco)      26 976         215           –      0      γ
                                Blanco        8829          70           –      0      δ
                               Turnout   9 442 977      75 398           –      –      τ
                            Electorate  12 524 152     100 000           –      –      λ
```

The simulation of Dutch Parliament

The routine DutchParliament[ ] generates the same result as the official assignment. It is not guaranteed that this will also be the case - or rather it is guaranteed that it will not always be so. The routine only includes the basic features and neglects some particulars. The routine helps to understand the issues and general mechanism and allows us to indicate consequences of alternatives.

```
Seats[] = official = Transpose[Rest[Drop[lis, -7]]][[5]]
```

```
{21, 30, 15, 31, 24, 10, 5, 10, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
dp = DutchParliament[]
```

```
{21, 30, 15, 31, 24, 10, 5, 10, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0}
```

The difference with the official results is:

```
dif = official - dp
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
? DutchParliament
```

DutchParliament[v, n] first sets Voters = v and NumberOfSeats = n, and then calls DutchParliament[]
DutchParliament[n] first sets NumberOfSeats = n, and then calls DutchParliament[]
DutchParliament[] uses the defaults already set
The routine first allots for ProperMajority and who gets at least one quota, then takes the Floor on the NaiveAssignment (for the remainder) and then applies HighestAverage (greatest average) on the remaining voters and seats. Option RemainingSeats might also be set to GreatestRemainder (for Dutch Councils with less than 19 seats) or SainteLagueWebster (for intellectual curiosity). Such application is to remaining seats and not from zero onwards

Global variables

For the simulations below it helps to explain some technical points. The routines use some global variables that make interactive use easier. The default values have been taken from these Dutch elections. The term “vote” can be ambiguous since citizens can vote for parties and parties can vote in Parliament; it is useful to say “(number of) voters” and “(number of) seats”. We use NVoters[ ] as a function (sum of Voters) and NumberOfSeats as a parameter.

```
? NumberOfSeats
```

Number of seats to be allocated. Must be set for routines to work. Default value 150 (Dutch Parliament)

```
? Seats
```

Seats[] can be set to the list of seats, with length NumberOfParties, and sum NumberOfSeats

```
? Voters
```

List of voters allocated across the parties. Must be set for routines to work.
Default value given by the Dutch Parliamentary elections in 2006, www.kiesraad.nl

From these two data we can find the number of voters required to gain a seat.

```
? ElectoralQuota
```

ElectoralQuota[] := NVoters[] / NumberOfSeats, i.e. the number of voters required to gain a seat
ElectoralQuota[Less] gives a list of 1 or 0 whether the party in Voters has less than the quota
ElectoralQuota[GreaterEqual] gives a list of 1 or 0 whether the party in Voters has at least the quota
ElectoralQuota[Less, Message] gives a message how many parties are below the quota
ElectoralQuota[Less, Test (, Message)] returns True when there are such parties otherwise False
ElectoralQuota[Assignment, a] gives a message (and False) when a is to a party below the quota

```
ElectoralQuota[]  // N
```

```
62 773.3
```

The wasted vote

```
? WastedVote
```

```
WastedVote is an option to routines like QualifiedThreshold
WastedVote[] can be set by the user to a value standing for the blank voters
    or (otherwise) invalid votes (and perhaps the non−voters). Typically WastedVote[N]
    for the number of voters and WastedVote[Seats] for the potential impact
WastedVote[0] is used in VoidSeats[Equations]
```

The notion of “wasted vote” derives from the context of proportionality. In a simple district method, a lot of views will not be represented but this is considered part of the system.

It is a design question to work with the long lists of original data or to clean them up and use the three sources of waste in a separate account. In some respects this can be immaterial since there can be cases with long lists anyway. We will use the two formats. The main point is to be aware of the denominator that determines the electoral quota.

The first source of waste are the blanco and invalid votes. More conventionally they are not included but it is fair to include them, so that they would affect the quota. In the Dutch data the number is small but it makes a difference of going from 1 to 2 wasted seats.

```
WastedVote[N] = 26976;
```

A second source of waste is from parties below the threshold from the electoral quota. They will not get seats but in principle they should be in the calculation since they contribute to the electoral quota.

A third source is the remainder = v - $q\,fna$, with v = Voters, q = the electoral quota, and fna = Floor[v / q] the floor of the naively assigned seats (i.e. v / q rounded down to an integer value). In Holland 2010 these are 6 seats and in current practice those voters are assigned to other parties than they voted for. Remaining seats in Holland are assigned using the method of highest average (that favours bigger parties) instead of the method of greatest remainder or the principle of Sainte-Laguë & Webster. As a result we get the surplus = v - $q\ s$, with s the officially assigned seats, and that surplus is negative for parties that gain and positive for parties that contribute. Is this mere mathematical approximation or are there political principles involved ?

```
surplus = Voters - Floor[Seats[] ElectoralQuota[]]
```

```
{−36354, −34395, −16904, −16398, −52067, 363, −8772,
  26434, −3229, 38035, 2010, 52937, 26196, 1255, 2042, 7456, 10471, 924}
```

The three sources of waste can be treated as one wishes. The three main methods are as follows (with the numbers of seats involved). The treatment by Dutch Parliament has been mentioned. The pure threshold assigns all non-quota votes to empty seats, thus 8 in total. The qualified majority threshold method uses this void to determine the qualified majority decision making threshold f > 1/2 and then distributes those 8 seats using Sainte-Laguë & Webster. We thus should distinguish the mirroring of a majority and the mirroring of the proportions in the vote (and it is not quite true that the latter takes care of the first).

```
     Method          Invalid       v < q                Remainder
      Seats            0.4          1.6                     6
 DutchParliament        No         Remove            Highest averages
  PureThreshold        Yes         To void                To void
ualifiedThreshold      Yes      For qual.maj.    (1) For qual.maj. (2) SL.
```

With these the essentials, let us look into the building blocks of proportionality.

3. The wasted vote

■ The Pure Threshold method and the data

Above we mentioned the pure threshold and the qualified majority threshold method. Before discussing them in detail it is useful to consider the data again. Let us first include the invalid votes and secondly clean up for the wasted parties below the threshold. The naive assignment means 2 seats for that total. Thirdly we can collect all remainders and find another 6 seats. Thus in total there are 8 empty seats. Simply leaving those empty can be called the pure threshold method.

```
MultipleSeatsCase[Set, 312 010];
pth = PureThreshold[WastedVote → WastedVote[N]];
```

Table: Comparison of official Dutch Parliament with the pure threshold assignment.

```
RemainderTable[Seats[], pth]
```

	Voters	Seats[1]	Remainder[1]	SeatFraction[1]	Diff.	Seats[2]	Remainder[2]	SeatFraction[2]
A	1 281 886	21	−40 130	−0.637	−1	20	22 823	0.363
B	1 848 805	30	−39 790	−0.632	−1	29	23 163	0.368
C	924 696	15	−19 601	−0.311	−1	14	43 352	0.689
D	1 929 575	31	−21 973	−0.349	−1	30	40 980	0.651
E	1 454 493	24	−56 383	−0.896	−1	23	6570	0.104
F	628 096	10	−1435	−0.023	−1	9	61 518	0.977
G	305 094	5	−9671	−0.154	−1	4	53 282	0.846
H	654 167	10	24 636	0.391	0	10	24 636	0.391
I	122 317	2	−3589	−0.057	−1	1	59 364	0.943
J	163 581	2	37 675	0.598	0	2	37 675	0.598
Void	130 267	0	130 267	2.069	8	8	−373 358	−5.931
Total	9 442 977	150	6	−0.001	0	150	5	−0.001

A drawback of empty seats is that people in a meeting may forget about them and simply fall back to the majority in the room. You don't tend to miss people who are never there. A tradition to always check on a qualified majority threshold would then be better. PM. The option of empty seats is implied by Mueller (1989:219) but there not elaborated on.

■ Principles of the Qualified (Majority) Threshold method

Proportional representation is a container concept for various approaches. Our study of the Dutch system showed some troubling properties. For political parties we would also require:

(1) A check on the majority principle.

(2) Inclusion of the blanco / invalid vote. Hence void seats or a qualified majority (that affects (1)).

(3) No assignment of voters to parties they did not vote for. Hence the remainder must give void seats or a qualified majority (that affects (1)).

(4) The method of Sainte-Laguë & Webster then is highly relevant for the adjusted data. The starting point is not zero but uses the assignment from (1) to (3). The empty seats can be used to better approximate the proportions in the non-empty seats.

This can be called the “qualified (majority) threshold” method. We thus distinguish representation and majority from the mere numerical exercise of approximating a distribution. NB. The non-voters are not included yet. Numerically they can easily be included in the routines but conceptually that application requires more research. Including them and raising the qualified majority might give too much of a bonus to absenteeism and we should rather stimulate voters to participate and express their opinion, even if it is a blanco vote.

The major unresolved issue with respect to these principles is that a coalition formed after the elections may get a majority in the electorate but not within Parliament with this qualified majority threshold. A coalition-dependent correction would involve a reassignment of seats at the cost of the non-coalition. The pure threshold method remains conceptually superior unless we formulate an additional criterion for coalitions. PM. A conceivable approach is to also require that the capacity to form coalities is copied from the electoral outcome to Parliament (Penrose-Banzhaf index).

■ Application

We would like to see that an assignment routine puts out a simple list but qualified majority is too important to hide in a background parameter. The output gives qualified majority as the integer fraction, the real and the number of seats. Including the blanco and invalid votes now raises the threshold to 80 seats.

```
MultipleSeatsCase[Set, 312 010];
qth = QualifiedThreshold[WastedVote → WastedVote[N], Hold → False]
```

$$\left\{\text{Seats} \to \{21, 30, 15, 31, 23, 10, 5, 10, 2, 3\}, \text{QualifiedMajority} \to \left\{\frac{75}{142}, 0.528169, 80\right\}\right\}$$

Table: Comparison of official Dutch Parliament with the qualified threshold assignment.

```
RemainderTable[Seats[], Seats /. qth]
```

	Voters	Seats[1]	Remainder[1]	SeatFraction[1]	Diff.	Seats[2]	Remainder[2]	SeatFraction[2]
A	1 281 886	21	−21 893	−0.353	0	21	−21 893	−0.353
B	1 848 805	30	−13 737	−0.221	0	30	−13 737	−0.221
C	924 696	15	−6575	−0.106	0	15	−6575	−0.106
D	1 929 575	31	4949	0.08	0	31	4949	0.08
E	1 454 493	24	−35 540	−0.572	−1	23	26 545	0.428
F	628 096	10	7249	0.117	0	10	7249	0.117
G	305 094	5	−5329	−0.086	0	5	−5329	−0.086
H	654 167	10	33 320	0.537	0	10	33 320	0.537
I	122 317	2	−1852	−0.03	0	2	−1852	−0.03
J	163 581	2	39 412	0.635	1	3	−22 673	−0.365
Total	9 312 710	150	4	0.001	0	150	4	0.001

PM 1. With the Hold → True option (default) the original list of parties can be maintained and the lists of Voters is not affected. With the application of Hold → False the list of parties is reduced to those that pass the quota threshold. Repeated application then causes repeated inclusion of the wastes vote and results in different outcomes. PM 2. The routine has stored the results in this place.

```
Options[QualifiedMajority]
```

$$\left\{\text{Fraction} \to \frac{75}{142}, \text{Real} \to 0.528169, \text{Integer} \to 80, \text{Seats} \to 150, \text{VoidSeats} \to 8\right\}$$