Meta-cognitive skills: Implementing Polya "How to solve it"

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1. Introduction

The following discussion has been inspired by the research done by Plonie Nijhof, Joris Ghysels and Rodica Ernst-Militaru. [2] [3]

The research appears to be an implementation of Polya (1945) "How to solve it". [4]

The Nijhof et al. research has earned the NRO prize for 2016. [5] [6] English slides of their approach are online from a ResearchED conference 2016, with the "META triangle" copied below in the **Appendix**. [7] I did not find a research paper by them yet.

I met Nijhof at the conference "Onderwijs meets Onderzoek" (June 20 2016) and there she told me about her research, which I deemed valuable then already. I decided to attend her workshop at the NVvW studyday (November 5 2016), and it was announced there that they had won that NRO 2016 prize. The workshop confirmed my impression that this research is valuable indeed. [8]

The workshop left me with some doubts about their choice of implementation. The following gives my comments and suggestions for improvements that can be tested. PM. CueThink also put out the four Polya steps in blocks on one page. [9] I will now focus on my comments that were inspired by the Nijhof et al. approach.

2. The main elements in the implementation

Polya distinguishes the four main steps: (1) understanding the question, (2) devising a strategy, (3) execution of the strategy, and (4) evaluation of the result. Awareness of these steps is part of the meta-cognitive skills: knowing about knowing. The steps are obviously relevant for all subjects but now we consider mathematics education.

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[1] Colignatus is the name in science of Thomas Cool, econometrician and teacher of mathematics, Scheveningen, Holland

[2] http://www.steunpuntopleidingsscholen.nl/wp-content/uploads/sites/2/2016/03/Poster-AOS-9-maart-2016-AOS-Passie-voor-leren.pdf

[3] https://www.nro.nl/kb/405-15-513-de-metadenkende-leerling-effecten-van-de-improve-methode

[4] http://www.maa.org/sites/default/files/Jeremy_Kilpatrick52328.pdf

[5] https://www.maastrichtuniversity.nl/news/mebit-alumna-rodica-ernst-wins-nro-education-prize-2016

[6] https://www.nro.nl/prijs/genomineerden-2016/rodica-ernst

[7] http://www.researched.org.uk/sessions/plonie-nijhof-rodica-ernst-militaru

[8] https://www.nvvw.nl/24089/subthema-c

[9] CueThink, CueNotes Ep.4 https://static1.squarespace.com/static/51606611e4b0b72aa94aed90/t/54d28190e4b02d784c7c9bef/1423081872745/CueNote4.pdf

These steps can be emphasized in teaching by pre-printing the steps in charts. See the **footnotes** for links to the different types of charts.

- Teacher and students have an *explanatory chart* that explains the steps and their subelements. [10] [11]
- Subsequently there are blank charts for further use. [12] [13]
    - ♦ The teacher can fill in one chart with supportive suggestions for a particular problem.
    - ♦ Students get an empty chart and fill in their perceptions.
- Students get a blank answer sheet (or screen) to answer the question in traditional manner. This answer sheet is graded. They may also use the blank charts as guidance, making changes in their own charts along the way. Their charts remain ungraded.

Potentially, there might be additonal training on using the charts. One would have classroom discussions about their actual use in solving problems. One might start with small example cases, and only present the explanatory chart once students have seen by example how it works (bottom up rather than top down).

Supposedly it works best to only offer the charts as guidance, and leave it up to students whether they show an interest and actually use them. A good explanation for this situation is that there is no standardized manner for grading the use of these charts. Once students notice that others have success by using the charts, success sells.

Nijhof et al. report that the discussion between teacher and students changes. When students grow aware about the steps being taken, they also adapt to the language used for those steps. Other students then might feel that they must study the charts themselves too if they want to benefit from the discussion.

3. *Comments that cause differences*

The above **footnotes** link to charts that I have designed as inspired by this research by Nijhof et al. I cannot present their work because it has a copyright statement (for "META-card").

I have the following comments that cause differences.

Nr.	*Included implementation of Polya 1945*	*Nijhof et al. 2016*
	Creative Commons BY-NC-SA 3.0 NL [14]	Copyright by META-card [15]
1	Use of English	They use Dutch but might already have an English version. [16]
2	The *explanatory chart* refers to Polya (1945) "How to solve it". This is not necessary for the empty chart or the charts created by filling in the blocks.	Their research refers to Polya but not their charts.
3	Only one version: blocks.	Two versions: blocks and triangle [17]
4	Polya's four steps are better presented as a cycle, like the empirical cycle. [18]	Blocks: Left to right, no arrows. Triangle with arrows: There is no clear reason why this would have to

[10] http://thomascool.eu/Papers/Math/Polya/2016-11-15-Polya-explanatory-chart.pdf
[11] http://thomascool.eu/Papers/Math/Polya/2016-11-15-Polya-explanatory-chart.doc
[12] http://thomascool.eu/Papers/Math/Polya/2016-11-15-Polya-blank-chart.pdf
[13] http://thomascool.eu/Papers/Math/Polya/2016-11-15-Polya-blank-chart.doc
[14] Copyright Thomas Cool 2016, https://creativecommons.org/licenses/by-nc-sa/3.0/nl
[15] I didn't have time to ask them why they put their copyright statement there, but since they do so, it seems best to insert my own now too. The left column clearly differs from the right column, with Polya (1945) as the common factor.
[16] The triangle in the **Appendix** is in English in slide 10: http://www.researched.org.uk/sessions/plonie-nijhof-rodica-ernst-militaru
[17] Perhaps triangle for questions and blocks for answers ?

		be a triangle. The triangles have less space for writing. The arrows are confusingly big and small.
5	Text is black on white with good contrast. Colour coding is only used for the border. (Keep the choice of tertiary colours for a quiet image.)	Colours in the background, so that the contrast with the text forms a problem. Their choice of tertiary colours indeed causes a quiet image.
6	Each step also has an icon. These are stylized and non-distractive, but students can adapt and make it "their own" (though keep the same colour codes). [19]	Colour codes only
7	Within the four steps, Polya's suggestions have been edited a bit: sorted differently, and with inclusion of new notions of mindmap and dynamic simulation. There is a closer link to mathematics and the distinction in steps of definitions, givens and proof. More prominence is given to the integration of text, formula, graph, numerical table (and dynamic simulation). [20]	Does not apply. PM. In the META approach mindmapping is important. [21]
8	Step 2 is called by some "Make a plan" or "Devise a strategy" and I turned this into "Devise a strategy and make a plan". While Polya is much about strategy, this is not entirely the same as an actual implemention of a strategy (a plan).	Use of "strategy" only

4. Relation to Bloom's taxonomy and "Mathematical think-activities" (MTA)

Alongside George Polya there is Bejamin Bloom and his taxonomy. [22]

- Nijhof et al. refer to Mevarech, who was a student of Bloom.
- There is the approach of "mathematical think-activities" (MTA). [23] The literature on MTA refers to Bloom too. It appears that MTA is misguided w.r.t. testing and the role of contexts (see the former footnote). [24]

[18] https://en.wikipedia.org/wiki/Empirical_research#Empirical_cycle
[19] Some students might be put off by the colour codes and icons, as I was myself hesitant, when I saw the colour codes by Nijhof et al. for the first time. However, after getting used to this, I agree that the colour coding can help students to distinguish the steps. The icons remain important however, not only for colour-blindness. Obviously, everyone should be aware that the steps are not 100% separable.
[20] Polya (1945) wrote before the advent of the ideology (not science) of "realistic mathematics education" (RME). A first step in understanding a problem might be to think about a concrete example. Nowadays, perhaps, a "context" may already have been provided in the problem question itself. We should distinguish however between ideology and science, see http://thomascool.eu/Papers/Math/2016-10-31-MTA.pdf
[21] I am not aware of a study that shows how mindmaps can improve results. If they would be, then one would hope for a lesson plan on how to create useful mindmaps.
[22] https://en.wikipedia.org/wiki/Bloom%27s_taxonomy
[23] http://thomascool.eu/Papers/Math/2016-10-31-MTA.pdf
[24] MTA has become an element in the Dutch standards for the national exams for graduation. While Polya has a distinction between routine and non-routine problems, my view is that a graduation exam must focus on routine questions that students can prepare for. It is invalid to have non-routine questions at the final examination. Non-routine questions are a teaching method. They are not suitable for graduation exams.

Nijhof et al. refer to the Motivated Strategies for Learning Questionnaire (MSLQ), [25] [26] Pintrich et al. (1991). [27]

Meta-cognitive insight should show up in direct test results (say for mathematics). Subsequently, such test improvement might be linked to scores on the MSLQ. Potentially 20% of the insights might be relevant for 80% of the improvements.

There is this [28] Dutch application of the MSLQ w.r.t. the development of talent.

Ben Wilbrink has a page on psychology that mentions both Polya and Bloom . [29]

5. *Some other resources*

I haven't actually studied Polya in the original text (shame on me), and only follow the summary reviews that I have heard over the years.

I have neither looked deeply at other efforts at implementing Polya. [30] [31] [32] [33] [34] [35] [36] Nijhof et al. refer to the Mevarech & Kramarski 1997 "IMPROVE" method [37] [38] [39] (see also sites of these authors). A quote:

> "And their findings are borne out by Robert Slavin at the UK Institute for Effective Education whose Best Evidence Encyclopedia (BEE) acknowledges IMPROVE as one of only two programs developed during the last 30 years that have demonstrably improved teenager's mathematical abilities." [40]

Apparently there is also Roger Azevedo & Vincent Aleven (2013), "International Handbook of Metacognition and Learning Technologies", available for $629. [41]

NRO has had also other projects, in Dutch. [42] [43]

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[25] https://www.nro.nl/kb/405-15-513-de-metadenkende-leerling-effecten-van-de-improve-methode
[26] http://www.indiana.edu/~p540alex/MSLQ.pdf
[27] http://files.eric.ed.gov/fulltext/ED338122.pdf
[28] https://www.leraar24.nl/api/publication/5445/file
[29] http://benwilbrink.nl/projecten/psychologie.htm
[30] Heppner, P. P., & Petersen, C. H. (1982), Problem Solving Inventory (PSI), http://www.sjdm.org/dmidi/Problem_Solving_Inventory.html
[31] Howard C. McAllister (1996), http://courses.cs.vt.edu/~cs1104/ProblemSolving/Polya/why1Polya.html
[32] Gerald L. Alexanderson (2000), "The Random Walks of George Polya", https://books.google.nl/books?id=OuHrR_6WEKsC&pg=PA235&lpg=PA235&dq=implementation+polya&source=bl&ots=De_3N8P1yx&sig=OClmhwlhk0M3jJMtgRXK6OKweXk&hl=en&sa=X&ved=0ahUKEwibnKHp16zQAhUF2xoKHSG8BrYQ6AEIMTAD#v=onepage&q=implementation%20polya&f=false
[33] Akhsanul In'am (2014), http://www.ccsenet.org/journal/index.php/ies/article/view/38219
[34] Hamid Arifin (2014), http://eprints.ums.ac.id/31108/12/9RR._NASKAH_PUBLIKASI.pdf
[35] Indika Wickramasinghe (2015), http://scholarcommons.usf.edu/cgi/viewcontent.cgi?article=1155&context=numeracy
[36] Behnoush Taheri et al. (2015), http://ijbpas.com/pdf/2015/December/1452615944MS%20IJBPAS%202015%20DEC%20SPCL%201068.pdf
[37] http://aer.sagepub.com/content/34/2/365.abstract
[38] http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG8/TG8_Kramarski_cerme3.pdf
[39] http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.670.3415&rep=rep1&type=pdf
[40] http://www.preventionaction.org/what-works/improve-math-first-question-self/1067
[41] http://link.springer.com/book/10.1007%2F978-1-4419-5546-3

There are other meta-cognitive skills, like the awareness of the Van Hiele levels of insight. The type of question and strategy depends upon the achieved level of insight. [44]

For mathematics, there are theorem proving skills that differ from other subjects. There was too much emphasis on these in the past, but the present puts not enough emphasis on these.

There can be a change for the whole curriculum when all subjects pay more attention to meta-cognitive skills. (One element is collaboration: allocate tasks to specialists.)

Teachers already know about the role of "student language", i.e. how students explain issues to each other. One aspect is that the teacher is at the highest Van Hiele level, and may have little access anymore to the situation of a student at a lower level, who still has to discover so much. Thus, also, students of higher grades might explain issues to students of lower grades. We already see at universities that graduates teach to undergraduates. We might adopt a similar approach in the schools. Teachings is a good way for students to learn (about meta-cognition). Obviously we would have exams on "minor teaching qualifications".

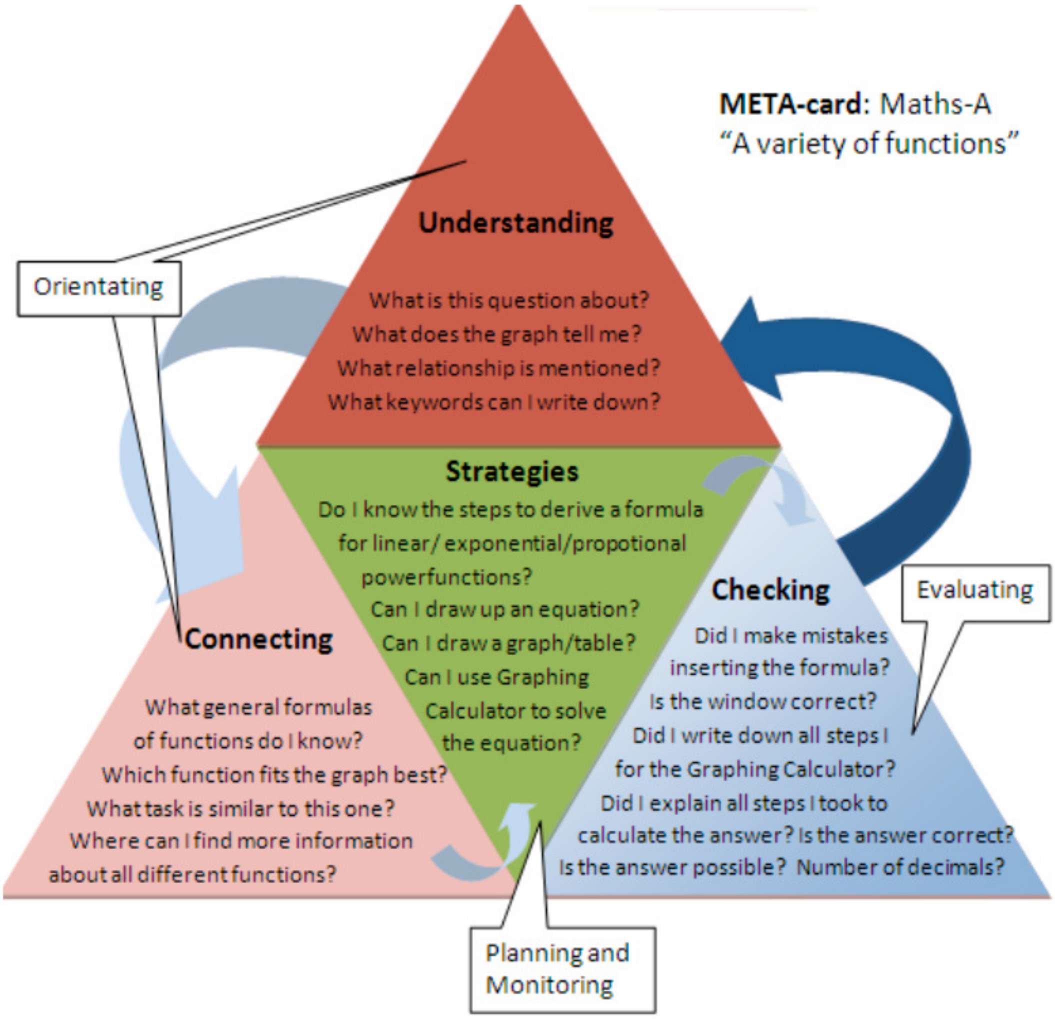
6. Concluding remarks

The charts that come along with this memo are only a suggestion and have not been tested. Actual tests will depend upon actual questions, say particular questions in geometry or trigonometry. Thus, we need a large number of tests to distinguish the different aspects: particulars in such questions and the common effects of metacognition, and the learning process over time itself.

[42] https://www.nro.nl/kb/405-14-532-reviewstudie-metacognitie-en-zelfgestuurd-leren
[43] https://www.nro.nl/kb/odb08002-hoogbegaafdheid-en-metacognitie-van-vwo-leerlingen-onderwijsbewijs
[44] Ben Wilbrink maltreats Van Hiele's work, see http://thomascool.eu/Papers/Math/2015-09-15-Breach-by-Jan-van-de-Craats-and-Ben-Wilbrink-wrt-scientific-integrity.html

Appendix. The Nijhof et al. META triangle

The following has been copied from slide 10 of the Nijhof et al. ResearchED conference 2016.[45]



[45] http://www.researched.org.uk/sessions/plonie-nijhof-rodica-ernst-militaru