Report on the paper
"An algebraic approach to the derivative"

by Thomas Colignatus

The author reviews his 'new approach to the derivative' developed in a series of works quoted at the end of the manuscript. The main part of the paper, however, is devoted to his views on division. According to the author, one should distinguish between the outcome of a division and the process of division, for which he proposes the notations x/y and $x//y$ respectively. The discussion is rather confused: the mathematical notion of division is confused with certain algorithmic aspects related how to determine the quotient in specific instances. The author seems to miss the point that, in a commutative ring R, the statement "$a/b = c$" is just another way of saying that $a = b \cdot c$. For instance, in the ring $\mathbb{Z}[x]$ of polynomials with integer coefficients over the variable x, the statement

$$\frac{x^2 - 1}{x - 1} = x + 1$$

just means that the following identity holds in $\mathbb{Z}[x]$:

$$x^2 - 1 = (x + 1) \cdot (x - 1).$$

This has absolutely nothing to do with the question whether $x - 1$ should be allowed or disallowed to be 0.

The section about derivatives is only half a page long and tells us to define the derivative $f'(x)$ as "$\Delta f//\Delta x$, then set $\Delta x = 0$." This may work fine if f is some simple polynomial, and obviously gives the correct result in that case, but the referee has no idea how to apply the recipe to, for instance, the function $f(x) = \sin x$. The process of dividing $\Delta \sin x$ by Δx leads nowhere, since it is not clear how to proceed from $(\sin(x + \Delta) - \sin x)//\Delta x$.

The section is concluded with some remarks about the derivative of the function $f(x) = |x|$. Here, the author fails to distinguish the notions of differentiability (this particular function is differentiable at every point $x \neq 0$) and the derivative (which equals $\mathrm{sgn}(x)$ at those points).

Coming to a conclusion, in the referee's opinion this paper is not suitable for publication.