

To Mastermath  
cc Organisations and persons involved

January 8 2017

Concerning: Your training programme for "Zij-instromers" for teacher of mathematics

Dear Sir, Madam,

Below, I attach a memo with some remarks w.r.t. your training programme for "Zij-instromers" who want to enroll in later training for teacher of mathematics. The reason for this memo is that I may need to report aspects to you that other people will not inform you about.

I am not personally involved in this programme. <sup>1</sup>

My comments in this memo are based upon my knowledge of *mathematics* <sup>2</sup> and *didactics of mathematics* <sup>3</sup> and the *situation w.r.t. teaching*. <sup>4</sup>

A major finding of my book "*Elegance with Substance*" (2009, 2015) <sup>3</sup> is that (i) mathematicians are trained for *abstraction*, (ii) while *teaching is empirical* and (iii) *didactics is an empirical science*. Thus teachers of mathematics should have a firm base in empirical science with added abstraction. The current model for training teachers appears *empirically* to be counterproductive, by first requiring a degree for (research) mathematics and only then adding some additional training for teaching.

The main comment for you and the "zij-instromers" is:

- "Zij-instromers" may require a raising of the level of mathematics indeed,
- but they have chosen not to become (research) mathematicians, neither in the original study nor in the current switch to teaching,
- and thus they, and their future students, are best served by raising the level of mathematics *with a focus on the requirements by didactics of mathematics*.
- However, you and own teacher trainers may have little knowledge about didactics of mathematics.
- My diagnosis is that you are currently making quite a mess on what I looked at.

I will put this letter and memo on my website, anonimised where relevant.

Kind regards,

Thomas Cool / Thomas Colignatus  
Econometrician (Groningen 1982) and teacher of mathematics (Leiden 2008)

Scheveningen

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<sup>1</sup> My personal history also differs from (most) current "zij-instromers". My degree in econometrics in Groningen 1982 was at an "interfaculteit" with equal involvement by the department of mathematics. Thus in Leiden 2008 I only needed courses on didactics of mathematics and pedagogy to complete the degree on teacher of mathematics. (Without those courses I already am and was fully qualified for teacher of economics too.)

<sup>2</sup> Article: "What a mathematician might wish to know about my work" at <http://www.thomascool.eu/Papers/Math/2013-03-26-WAMMWTKAMW.pdf>

<sup>3</sup> "Elegance with Substance", pdf online, <http://thomascool.eu/Papers/Math/Index.html>

<sup>4</sup> <http://thomascool.eu/Papers/Math/2016-06-28-Letter-to-NVVW-with-Red-Card.pdf>

# Some remarks about the Mastermath 2017 training programme for "Zij-instromers" for teacher of mathematics

Thomas Colignatus  
January 8 2017

1.	Introduction .....	3
1.1.	Outline .....	3
1.2.	Concepts: Mastermath and "zij-instromers" .....	3
1.3.	Major conclusions of "Elegance with Substance" (2009, 2015) .....	3
1.4.	Letter to VOR and Institutes for Training of Teachers .....	4
1.5.	Focus on "zij-instromers", and main concerns in this memo .....	5
1.6.	Caveat: Looked at only " <i>Foundations</i> " and " <i>Analysis</i> " .....	5
1.7.	PM. Explanatory quote in English .....	6
1.8.	PM. Explanatory quote in Dutch .....	6
2.	Highschool mathematics versus research mathematics .....	6
2.1.	Key example: Real numbers, continuity and limits, and group theory .....	6
2.2.	Costs in education by requiring limits for the derivative .....	8
2.3.	Math war in Holland .....	10
2.4.	Costs of the math war .....	11
3.	On "Analysis" .....	11
3.1.	Use a standard book and not your own undidactic syllabus .....	11
3.2.	W.r.t. "Conquest of the Plane" (COTP) (2011) .....	12
4.	On "Foundations" .....	12
4.1.	Logic .....	12
4.2.	ZFC .....	13
4.3.	Group theory .....	13
4.4.	"Fractions" .....	15
4.5.	From numbers to neighbourhoods .....	15
4.6.	Complex numbers and linear algebra .....	16
5.	Qualifications of the Mastermath teacher trainers themselves .....	16
6.	Conclusions .....	16
	Appendix. Letter to Wiegerinck w.r.t. Mastermath course in Analysis .....	17

## 1. Introduction

### 1.1. Outline

This *Introduction* will clarify what Mastermath states as its purpose. It will also give a summary of the major conclusions of "Elegance with Substance" (2009, 2015) and of this memo w.r.t. the training of potential teachers.

Subsequently, I will make some comments on didactics, in relation to the Mastermath courses on "*Foundations*" and "*Analysis*".

It is unavoidable to also consider the qualifications of the Mastermath teacher-trainers themselves. They focus on teaching mathematics (the stated purpose) but it would be better if they knew about didactics and teaching at highschool.

We close with conclusions.

### 1.2. Concepts: Mastermath and "zij-instromers"

Mastermath is the master's degree programme in (research) *mathematics*, in The Netherlands (Holland).

Mastermath also has a "remedial" programme for potential *teachers of mathematics*, for people who have an original degree in science but no degree in mathematics. These people are called "zij-instromers".

The aim is to get these "zij-instromers" qualified for the official teacher of mathematics training programmes. There are 7 courses of each 6 ECTS, or 7 x 6 x 28 hours or 42 weeks of 28 hours studying (assuming also other activities).

Currently, there are about 5 persons taking this course. <sup>5</sup>

New website (2016-2017): <http://elo.Mastermath.nl>

The website for Spring 2017: <https://elo.mastermath.nl/course/index.php?categoryid=17>

(Old website: <http://www.Mastermath.nl>)

### 1.3. Major conclusions of "Elegance with Substance" (2009, 2015)

"*Elegance with Substance*" (2009, 2015) <sup>6</sup> has more conclusions, but major ones for this topic are:

- Mathematicians are trained for abstraction while teaching is empirics, and didactics an empirical science.
- Thus teachers of mathematics should have a firm base in empirical science with added abstraction.
- The current approach to training teachers of mathematics is counterproductive, as *empirical evidence shows*, since it first requires a degree for (research) mathematics and only then adds some additional training for didactics and teaching.

Thus, also the current required qualifications for the official teacher of mathematics training programmes should be revised.

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<sup>5</sup> <http://www.beteronderwijsnederland.nl/content/mastermath-voor-zij-instromers-wiskunde>

<sup>6</sup> <http://thomascool.eu/Papers/Math/Index.html>

## 1.4. Letter to VOR and Institutes for Training of Teachers

It is important to be aware of the general environment. When "zij-instromers" are prepared for the regular teacher training courses, then it is useful to be aware what these training courses for teacher of mathematics amount to.

(1) This is my letter <sup>7</sup> and sequel <sup>8</sup> to the *association for research in education* (VOR) <sup>9</sup> and the *institutes for training of teachers* (ICL). <sup>10</sup> I haven't received a reply yet.

(2) A recent report by NVAO about the university training for teachers is here. <sup>11</sup> On page 18, there is the observation that the training has a double objective, both the masters degree itself and the training for teacher. This is called a "challenge", but, if this report had looked at *"Elegance with Substance"* (2009, 2015), then they could have concluded that for mathematics and its teaching it is a "logically inconsistent" (both empirics and neglect of empirics by abstraction):

### *Dubbele opdracht van de ulo's*

Er is een spanning tussen de voorbereiding op het beroep van docent en de eisen die worden gesteld aan een opleiding op masterniveau. Het lerarenberoep vraagt niet alleen cognitieve vaardigheden maar ook communicatieve, pedagogische en didactische vaardigheden: vaardigheden die nodig zijn om een goede relatie op te kunnen bouwen met leerlingen en ingewikkelde leerstof goed uit te kunnen leggen. Dit zijn vaardigheden waarvan de ontwikkeling tijd kost en die ulo-studenten in hun voorafgaande vakbachelor of –master veelal niet hebben ontwikkeld. Tegelijkertijd dient de opleiding wel op masterniveau te zijn, en daarmee een vergelijkbare onderzoekscomponent te kennen als andere masters. De ulo valt onder hetzelfde accreditatieregime als andere masteropleidingen. Dit stelt de ulo's voor een ingewikkelde opdracht, zeker als de opleidingsduur maar één jaar bedraagt.

(3) The Mastermath training programme for "zij-instromers" should be aware of literature for the training of teachers. For example, there is Wansink (1971), *"Didactische Oriëntatie voor Wiskundeleraren"*, parts I, II and III, Wolters-Noordhoff. Various parts might be outdated, but others are not, and their level of mathematics is quite decent. Thus Mastermath has ample scope for linking up with existing literature on didactics at decent level of mathematics, and there is less need for Mastermath teachers to try to invent the wheel themselves without adequate background in didactics themselves.

(4) It is useful to be aware that the Dutch association of teachers of mathematics (NVvW) is a seriously sick association. <sup>12 13</sup>

(5) It is useful to be aware that PWN submitted a misleading "Deltaplan Wiskunde" to the minister of Education. <sup>14 15</sup>

<sup>7</sup> <http://thomascool.eu/Papers/Math/2016-05-09-Letter-to-VOR-and-Trainers-of-teachers.pdf>

<sup>8</sup> <http://thomascool.eu/Papers/Math/2016-07-15-Second-letter-to-VOR-and-Trainers-of-teachers.pdf>

<sup>9</sup> <http://www.vorsite.nl/en>

<sup>10</sup> <http://www.universitairelerarenopleidingen.nl>

<sup>11</sup> <https://www.rijksoverheid.nl/documenten/rapporten/2016/11/24/van-verleden-naar-toekomst-de-universitaire-lerarenopleidingen>

<sup>12</sup> <http://thomascool.eu/Papers/Math/2016-06-28-Letter-to-NVvW-with-Red-Card.pdf>

<sup>13</sup> <http://thomascool.eu/Papers/AardigeGetallen/2017-01-06-Aan-staatssecretaris-Sander-Dekker-en-OCW.pdf>

<sup>14</sup> <http://thomascool.eu/Papers/AardigeGetallen/2016-03-22-Minister-krijgt-een-misleidend-Deltaplan-Wiskunde.pdf>

<sup>15</sup> <http://thomascool.eu/Papers/Math/2016-03-15-Some-experiences-with-HdS-RB-FvdDS-JvdC.html>

## 1.5. Focus on "zij-instromers", and main concerns in this memo

This memo focuses on the "zij-instromers". Main concerns are:

- (1) Students following these Mastermath courses should get quality mathematics.
- (2) These students didn't choose (research) mathematics as their original interest. Now they opt for teaching rather than (research) mathematics too. Thus, Mastermath cannot just present them with pure mathematics courses as for (young) students of mathematics.
- (3) It is possible to achieve a high level of mathematics *that has a focus on didactics of mathematics*. This would be an ideal combination: both quality of mathematics, and a preparation for teaching that these students are interested in.
- (4) However, didactics is an empirical science. Teacher-trainers at Mastermath have qualifications for mathematics but lack qualifications for teaching and/or empirical research in didactics of mathematics.
- (5) Thus there is currently a mismatch, that shows from the quality of the Mastermath course design, teacher-trainers and textbooks.
- (6) It would be possible to actually improve the programme, not only in theory but also in practice.

## 1.6. Caveat: Looked at only "Foundations" and "Analysis"

From the 7 "zij-instromer" courses, I only looked at:

- Fall 2016: "Foundations" ("Fundamenten"). This isn't quite "foundations of mathematics" but "basic concepts". Teachers are Bas Edixhoven<sup>16</sup> and Theo van den Bogaart<sup>17</sup> (who did his thesis with Edixhoven).  
[https://www.beteronderwijsnederland.nl/sites/beteronderwijsnederland.nl/files/fundamente\\_n.pdf](https://www.beteronderwijsnederland.nl/sites/beteronderwijsnederland.nl/files/fundamente_n.pdf)
- Spring 2017: "Analysis". Teachers are Joost Hulshof<sup>18</sup> and Jan Wiegerinck.<sup>19</sup>  
<https://www.beteronderwijsnederland.nl/sites/beteronderwijsnederland.nl/files/Analyse2017.pdf>

Thus, I did not look at:

- *Algebra and Number theory* (Wieb Bosma<sup>20</sup> and Jaap Top<sup>21</sup>)
- *Numerical methods and Optimisation* (Martijn Anthonissen<sup>22</sup> and Jan ten Thije Boonkkamp<sup>23</sup>)
- History (Jeanine Daems<sup>24</sup> and Steven Webster<sup>25</sup>)
- Geometry (Gerard Jeurnink,<sup>26</sup> Jeroen Spandaw<sup>27</sup> and Hans Sterk<sup>28</sup>)
- Stochastics (Eric Cator,<sup>29</sup> Cor Kraaikamp<sup>30</sup>)

<sup>16</sup> <http://www.math.leidenuniv.nl/~edix>

<sup>17</sup> <https://www.linkedin.com/in/theo-van-den-bogaart-79b78261> (highschool teaching in 2006-2009)

<sup>18</sup> <http://www.few.vu.nl/~jhulshof>

<sup>19</sup> <https://staff.science.uva.nl/j.j.o.wiegerinck>

<sup>20</sup> <https://nl.linkedin.com/in/wieb-bosma-66a11812>

<sup>21</sup> <http://www.math.rug.nl/~top/index.html>

<sup>22</sup> <https://www.tue.nl/universiteit/over-de-universiteit/eindhoven-school-of-education/over-esoe/medewerkers/detail/ep/e/d/ep-uid/199219241> (has been involved in teaching didactics of mathematics)

<sup>23</sup> <https://www.linkedin.com/in/jantenthije>

<sup>24</sup> <https://www.linkedin.com/in/jeaninedaems> (also a highschool teaching degree in 2011, teaching experience 2010-2013, teacher trainer since 2011)

<sup>25</sup> <https://www.uu.nl/staff/SAWepster> (no cv online)

<sup>26</sup> <https://www.utwente.nl/ewi/aa/people/personalpages/personalpage-jeurnink> (parttime involved in teacher training)

<sup>27</sup> <http://www.tnw.tudelft.nl/en/about-faculty/departments/other-research-groups/science-education-and-communication/people/spandaw-jg-dr> (teacher in 2003-2007, involved in teacher-training)

<sup>28</sup> <https://www.tue.nl/universiteit/over-de-universiteit/eindhoven-school-of-education/over-esoe/medewerkers/detail/ep/e/d/ep-uid/199219241/ep-tab/3> (also given courses on teacher training)

<sup>29</sup> <http://www.ru.nl/@864256/dr-eric-cator>

## 1.7. PM. Explanatory quote in English

QUOTE (without the "zij-instromer" teacher part)

The Departments of Mathematics of the Dutch universities have combined their efforts to enhance their master programmes in mathematics. Part of the cooperation is aimed at organising joint courses in mathematics. These joint courses offer students the highest quality of instruction and open up opportunities for interaction with students of other institutes of mathematics. In the master programme of each university it is stipulated how many credit points must be earned by taking courses in the national master programme. For students who intend to pursue a PhD programme after completing their master's programme, the national programme increases the range of options open to them. For further questions concerning the national master's degree programme in mathematics consult your local director of the master's programme.

UNQUOTE

## 1.8. PM. Explanatory quote in Dutch

QUOTE (my emphasis)

Het programma van zeven lerarenvakken is ontstaan uit een initiatief van Mastermath en de Nederlandse vakdidactici in de wiskunde om actie te ondernemen tegen het tekort aan eerstegraads docenten.

Het programma is primair gericht op studenten die een academische bètamasteropleiding anders dan wiskunde hebben afgerond. Door het volgen van een aantal van deze vakken kunnen zij voldoen aan de vakinhoudelijke **toelatingseisen** van de universitaire lerarenopleidingen. De te volgen vakken hangen af van de vooropleiding. De lerarenopleiding kan hierover informeren.

Daarnaast staat het pakket open voor:

- HBO-masterstudenten met een positief advies van hun opleiding;
- eerstegraadsdocenten wiskunde die hun kennis willen verdiepen en actualiseren, in de vorm van nascholing;
- studenten met een afgeronde academische alfa- of gamma-masteropleiding, die door middel van een "assessment" getoond hebben het pakket te kunnen volgen;
- studenten die een universitaire educatieve masteropleiding volgen (vanwege overlap met het bachelor wiskunde programma kan slechts een deel van het pakket meetellen, studieadviseur en examencommissie kunnen hierover informeren).

UNQUOTE

## 2. *Highschool mathematics versus research mathematics*

### 2.1. Key example: Real numbers, continuity and limits, and group theory

In mathematics, it is possible to define continuity by using limits, and to define limits by continuity. Both methods can be applied to the real numbers, with topology as "the science of infinite closeness without distance".<sup>31</sup>

A teacher of mathematics should know about the flexibility of concepts here, with the ability to choose a particular setup.

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<sup>30</sup> <http://www.ewi.tudelft.nl/en/the-faculty/departments/applied-mathematics/applied-probability/people/wetenschappelijke-staf/cor-kraaikamp>

<sup>31</sup> <https://staff.fnwi.uva.nl/j.vanmill>

However, for the highschool curriculum, it is better not to confuse students, and *present them with a clear line of reasoning*. The set of options better be reduced to a single choice for the highschool curriculum.

What would be that single choice ? My conclusion of 2015-2016 is that it is better to have a firm foundation in the real numbers (including issues of approximation), use this for the definition of continuity (of the continuum and for functions separately), and then define the notion of limit with this continuity.

Thus, for a teacher of mathematics it is required to have a firm foundation about a particular setup and line of reasoning (and hopefully the one above). It is useful to have an idea about other setups, but an idea only. It is up to research mathematicians to study alternative setups, but this should not confuse others.

NB. Also for mathematics itself, we should expect a clear line, and not a mere collection of concepts. Perhaps there are alternative lines, but not too many. For example, the Weierstrasz epsilon and delta would have the domain of real numbers (thus not just  $\forall \varepsilon \exists \delta$  but  $\forall \varepsilon \in \mathbb{R} \exists \delta \in \mathbb{R}$ ), and then this approach to the limit could not be used for creating particular aspects of the real numbers themselves, because of circularity in reasoning.

The "Foundations" course focuses on group theory, while it is much more important for teachers of mathematics to have a firm grip on notations, that are of key importance for students. The issue of notation is mathematically as least as challenging as group theory (though it might be closer to Informatics & Programming and not be appealing to the Group Theorists in the Mastermath team).<sup>32</sup>

Obviously, the curriculum setup to have the notation of the real numbers as the foundation resolves both the issues of continuity and confusions about group theory.

Relevant documents for this conceptual path for highschool are:

- "Foundations of Mathematics. A Neoclassical Approach to Infinity" (FMNAI) (2015).<sup>33</sup> This also argues for a "neoclassical" approach to didactics and curriculum.<sup>34</sup>
- Linking up to didactics discussions by Vredenduin, Van der Blij and Van Dormolen.<sup>35</sup>
- Supporting weblog text on the latter.<sup>36</sup>
- The algebraic approach to calculus can avoid limits. The proof of concept is "*Conquest of the Plane*" (COTP) (2011),<sup>37</sup> favourably reviewed by Gill (NAW)<sup>38</sup> and Gamboa (EMS),<sup>39</sup> with a protest by Boudri<sup>40</sup> against maltreatment by Spandaw.
- A recent review of the situation w.r.t. the algebraic approach.<sup>41</sup>
- A recent comparison to the algebraic method by Descartes.<sup>42</sup> (See Michael Range (2016), "*What is Calculus?: From Simple Algebra to Deep Analysis*".<sup>43</sup>)
- To repeat: a focus on notation rather than group theory.<sup>44</sup>

<sup>32</sup> <https://boycottholland.wordpress.com/2017/01/03/education-group-theory-and-division>

<sup>33</sup> <http://thomascool.eu/Papers/FMNAI/Index.html> (pdf online)

<sup>34</sup> <http://thomascool.eu/Papers/AardigeGetallen/2015-10-14-Rekenen-Fraude-Freudenthal-Parlement.pdf>

<sup>35</sup> <http://thomascool.eu/Papers/Math/2016-11-30-Algebraic-Approach-Derivative-and-Continuity.pdf>

<sup>36</sup> <https://boycottholland.wordpress.com/2016/11/28/definition-of-continuity-written-out>

<sup>37</sup> <http://thomascool.eu/Papers/COTP/Index.html>

<sup>38</sup> <http://www.nieuwarchief.nl/serie5/pdf/naw5-2012-13-1-064.pdf>

<sup>39</sup> <http://www.euro-math-soc.eu/review/conquest-plane>

<sup>40</sup> <http://thomascool.eu/Papers/COTP/2013-03-15-Boudri-over-COTP.pdf>

<sup>41</sup> <http://thomascool.eu/Papers/Math/2016-08-14-An-algebraic-approach-to-the-derivative.pdf>

<sup>42</sup> <https://boycottholland.wordpress.com/2016/12/26/descartes-wins-from-fermat-on-the-incline>

<sup>43</sup> <http://www.worldscientific.com/worldscibooks/10.1142/9448>

<sup>44</sup> <https://boycottholland.wordpress.com/2017/01/03/education-group-theory-and-division>

## 2.2. Costs in education by requiring limits for the derivative

When teachers of mathematics design highschool courses with the use of limits for the derivative, then teachers of physics meet with frustration.

A current physics textbook starts with a graphical discussion of place, speed and acceleration right at the beginning of class 4 of VWO in September. A standard mathematics textbook waits with the derivative till Spring next year of the same class. Thus there is an unfortunate gap of some half year, at a critical moment.

The comparison text for the curriculum on mathematics & physics<sup>45</sup> contains a claim that the derivative "requires" the limit (see page 28). *This is false claim*, see my book COTP<sup>46</sup> or the book by Michael Range<sup>47</sup> (and see the **Appendix**).

If mathematicians – and also you in your training position – keep on disinforming teachers of mathematics that limits are required, then weak teachers might opt to follow the mathematicians rather than follow empirical observations on didactics. You would be doing indoctrination without actual knowledge about the requirements for education. And you would not follow the ethics of mathematics to be truthful about the choices involved.

Michael Range states:<sup>48</sup>

**1. INTRODUCTION.** Have you ever wondered why we burden our students with limits when teaching about tangent lines and differentiation of rational, root, and similar algebraic functions? Of course, as experienced mathematicians, we know that limits ultimately cannot be avoided. However, the early emphasis on limits in the context of differentiation of numerous algebraic examples may cause quite a bit of confusion. After all, the calculation of such derivatives relies primarily on algebraic techniques to rewrite the difference quotient in such a way that one can cancel the troublesome  $h = \Delta x \neq 0$  from the denominator. The final answer then follows by what appears to be just *plugging in*  $h = 0$ . Surely most students, when first shown the derivative of  $y = x^2$ , are hard pressed to understand the subtlety of the statement  $\lim_{h \rightarrow 0}(2x + h) = 2x$ , a conceptual leap that took mathematicians close to two centuries to fully understand and to formulate correctly. Yes, we try to teach our students that we do need to take the *limit* as  $h \rightarrow 0$ , rather than just *evaluate* at  $h = 0$ . On the other hand, evaluating at  $h = 0$  is eventually justified by invoking the *continuity* of the relevant functions. No wonder today's students in a standard first calculus course typically retain little about limits. They have grown up with graphing calculators, and continuity—at least its intuitive geometric interpretation—looks obvious to them. Consequently it is difficult for them to grasp the need for limits as long as one considers only algebraic functions.

Thinking about these difficulties in the teaching of elementary calculus, I was somewhat surprised to discover that there is a very simple and natural algebraic approach to differentiation of algebraic functions that avoids limits altogether and justifies the students' "easy" calculation of derivatives by "plugging in." More surprising was the realization that this approach had not been used systematically in the early days of calculus, when mathematicians struggled unsuccessfully for nearly two centuries to resolve the inconsistencies and mysteries regarding infinitely small quantities, infinitesimals, and differentials that are zero or nonzero depending on what suits the purpose.<sup>1</sup> While the basic idea already appears in the work of René Descartes

<sup>45</sup> <http://downloads.slo.nl/Repository/afstemming-wiskunde-natuurkunde-tweede-fase.pdf>

<sup>46</sup> <http://thomascool.eu/Papers/COTP/Index.html>

<sup>47</sup> <http://www.worldscientific.com/worldscibooks/10.1142/9448#t=aboutBook>

<sup>48</sup> <http://www.maa.org/press/periodicals/american-mathematical-monthly/american-mathematical-monthly-contents-may-2011>



Shen & Lin (2014)<sup>49</sup> state:

hard mass formed by minerals. Obviously here we are interested in the first meaning. We wish to demonstrate that (i) the calculus method can be developed by analyzing steepness and height change of a curve, (ii) the method development can be achieved directly using Descartes' method of tangents and does not need an introduction of limit as prerequisite, and (iii) the basic method of calculus and a few simple examples can be introduced in a one-hour or two-hour lecture to a high-school level audience.

Calculus is one of the most important tools in a knowledge based society. Millions of people around the world learn calculus everyday. All engineering, science, and business major undergraduate students must take calculus. Many high schools offer calculus courses. The usefulness and power of calculus have been well recognized. Nonetheless, calculus is a mysterious subject to many people and is regarded by the general public as accessible only to a few privileged people with special talents. Tight schedules and high fail rates for the first semester calculus have given the course a reputation as a monster, a nightmare, or a psychological barrier for many students, some of whom are even STEM (science, technology, engineering and mathematics) majors. Calculus can be a topic that causes people at a social gathering to shake their heads in incomprehension, shy away from the daunting challenge of understanding it, or express effusive exclamation of awe and admiration. It is also sometimes associated with conspicuous nerdiness. In classrooms, the student-instructor relationship can be tense. Some students regard calculus instructors as inhuman and ruthless aliens, while instructors frequently joke about students' stupidity, clumsiness, or silly errors. Tedious and peculiar notations coupled with fiendish and complex approaches to calculus teaching and learning may have contributed to the above unfortunate situation.

A major cause of this mystery and scare of calculus is unnecessarily complex terminologies, including definite and indefinite integrals, derivatives defined as limits, definite integrals defined as limits, the difference between  $dx$  and  $\Delta x$ , and using ever-finer divisions of an area under a

PM. I refer to Range and Shen & Lin as quite relevant sources. However, they haven't yet incorporated my suggestion of the "dynamic quotient". It is the latter that also settles the issue for trigonometry and exponentials, see COTP (pdf online).<sup>50</sup>

We should distinguish (a) the design of a course for highschool and matricola for non-math majors and (b) mathematics itself. Obviously, math majors must know about the approach via limits too. My impression is that COTP also allows an essential refoundation of the derivative for mathematics itself too. However, I am no mathematician and don't have the desire of a career in mathematics, and I don't tend to pursue the latter line of (b). My focus is on (a).

The point remains that *education has a needless burden* with limits for the derivative, with the main responsibility for teachers, but apparently academic mathematicians being unaware of their responsibility when they teach mathematics to prospective teachers.

Let me add that Michael Range (2016:5)<sup>51</sup> has a definition of tangency (root with multiplicity of more than 1) that I find problematic. It seems to turn an effect of some cases (with factorisation) into a general cause. Let me refer to a later statement on this. Still, his book is a step to greater acceptance of the algebraic approach.

Let me add that Michael Range (2016:13-14) actually *uses* the steps for the "dynamic quotient" in COTP (2011:57)<sup>52</sup> but without reference to its formal development. This is no surprise, since the dynamic quotient has been developed to capture the practice in mathematics.<sup>53</sup> Yet, the major conceptual step for mathematicians is to accept the definition of the dynamic quotient, as hard as it can be to accept a definition. (Notably: it is important to grow aware that the formula for a function also contains key information.)

<sup>49</sup> <https://arxiv.org/abs/1404.0070>

<sup>50</sup> <http://thomascool.eu/Papers/COTP/Index.html>

<sup>51</sup> [http://www.worldscientific.com/doi/suppl/10.1142/9448/suppl\\_file/9448\\_chap01.pdf](http://www.worldscientific.com/doi/suppl/10.1142/9448/suppl_file/9448_chap01.pdf)

<sup>52</sup> <http://thomascool.eu/Papers/COTP/ConquestOfThePlane.pdf>

<sup>53</sup> <http://thomascool.eu/Papers/Math/2016-11-30-Algebraic-Approach-Derivative-and-Continuity.pdf>

Michael Range (2016:13-14):

by formula (P.2), since this formula now gives the meaningless expression  $\frac{0}{0}$ . However, if we rewrite the equation that defines velocity as the product  $distance = velocity \times time$ , then the problem becomes more manageable. In fact, let us consider the simple case considered by Galileo, i.e.,  $d(t) = ct^2$ . If we fix a particular time  $t_0$ , then  $d(t) - d(t_0) = ct^2 - ct_0^2$ , which factors into

$$d(t) - d(t_0) = c(t + t_0)(t - t_0). \quad (\text{P.3})$$

Note that if  $t > t_0$  the factor  $c(t + t_0)$  in this last formula obviously equals the average velocity over the time interval from  $t_0$  to  $t$ . (Just divide both velocity is allowed to be both positive and negative (or zero), with the sign accounting for the direction of motion along a line. More generally, when the motion is not constrained to a line, the velocity is represented by a so-called *vector*, a more complicated quantity that encodes, for example, the direction of the motion in space.

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14

*What is Calculus? From Simple Algebra to Deep Analysis*

sides of (P.3) by  $t - t_0 \neq 0$ .) This also holds if  $t < t_0$ , where the time interval now goes from  $t$  to  $t_0$ . (See Problem 2 of Exercise 4.1.) Therefore, trusting in the consistency of the formula (P.3), we are led to define the velocity at  $t_0$  by taking the value of this factor at  $t = t_0$ , i.e., we define

$$v(t_0) = c(t_0 + t_0) = 2ct_0.$$

Perhaps you have some doubts about the validity of this definition. After

### 2.3. Math war in Holland

Normally, I should not have to write to you about this. However, it appears that there is a math war in Holland, such that useful information is *misrepresented, abused, neglected or burked, with deliberate cheating and lying*.

My original advice in 2008 was that there should be a parliamentary enquiry into mathematics education, and this advice was based upon my analysis as an economist about the industry of mathematics and its education.

It is only later that I discovered this math war. I have been documenting this abuse since 2008, with the idea that this might be useful for parliament one day.

One can recognise these conceptual positions: <sup>54</sup>

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<sup>54</sup> <https://boycottholland.wordpress.com/2016/01/24/graphical-displays-about-the-math-war>

- The ideology of "realistic mathematics education" (RME), with the Freudenthal Head in the Clouds Realistic Mathematics Institute (FI = FHCRM).
- The ideology of "traditional mathematics education (TME), with the "Stichting Goed Rekenonderwijs" (SGR).
- My suggestion of "neoclassical mathematics education" (NME), which is a proposal to re-engineer mathematics education, the develop hypotheses that need to be tested is empirical research. It are the students who determine what works. (See this recent workshop at the NVvW annual meeting 2016.<sup>55</sup>)
- Most teachers who teach as they have been doing.

## 2.4. Costs of the math war

The math war has various costs.

A recent example is that Vos & Roorda presented empirical research on didactics of the derivative. They presented students with a statement: " $A(a)$  is the use of gasoline after  $a$  km." They didn't inform the students that  $A$  was a function. Thus students also had the option to eliminate brackets, so that the difference  $V(a + h) - V(a)$  reduces to  $Vh$ , and one cannot say that these students are wrong. My analysis in "Elegance with Substance" (2009, 2015) has been that either one always states that there is a function, or one uses other brackets, like  $A[a]$ , to indicate functions. But apparently Roorda has been burking my work, even though he knows about it.<sup>56 57 58</sup>

A recent case is that the board of NVvW claimed that there was an "agreement" with education secretary Dekker, while Dekker denies this. My analysis indicates a deliberate lie by the board of NVvW, likely to cover up their own errors since 2005.<sup>59 60</sup>

One cost of the math war is that Mastermath presents "zij-instromers" with a disinformative programme. For example, it would be useful when these prospective teachers of mathematics develop the habit of writing functions with square brackets too (that is, in the convention of *Mathematica* that apparently is without risk of confusion).

## 3. On "Analysis"

### 3.1. Use a standard book and not your own undidactic syllabus

On Analysis, the main points can be found in the **Appendix**, in Dutch, "Letter to Wiegerinck w.r.t. Mastermath course in Analysis".

Relevant points are:

- Mastermath is advised to use available textbooks, like Adams & Essex or Range, instead of writing your own syllabus, as now is being done by author Hulshof *who clearly lacks credentials as a writer*. The book by Range would be recommended, since one can easily enlighten critical points by reference to common material. (But *tell me what you think about my own exposition, in COTP.*)
- Mathematician and main author of the syllabus Hulshof might *seem* to know about didactics given his public statements, but he has no qualification for didactics at

<sup>55</sup> <http://thomascool.eu/Papers/AardigeGetallen/2016-11-03-Presentation-Math-Ed-Redesign.pdf>

<sup>56</sup> <http://thomascool.eu/Papers/AardigeGetallen/2016-12-21-Vos-Roorda-Afgeleide.pdf>

<sup>57</sup> <http://thomascool.eu/Papers/BHRM/2015-10-28-Malconduct-Roorda-Daemen-Drijvers.pdf>

<sup>58</sup> <http://thomascool.eu/Papers/AardigeGetallen/2015-10-30-Brief-aan-redactie-Epsilon-Uitgaven.html>

<sup>59</sup> [http://thomascool.eu/Papers/AardigeGetallen/2008-2016-plus-Afgewezen-door-de-Wiskunde-brief.html#2016-12-10\\_\(2\)](http://thomascool.eu/Papers/AardigeGetallen/2008-2016-plus-Afgewezen-door-de-Wiskunde-brief.html#2016-12-10_(2))

<sup>60</sup> <http://thomascool.eu/Papers/AardigeGetallen/2017-01-06-Aan-staatssecretaris-Sander-Dekker-en-OCW.pdf>

highschool level, and there would be much to comment on his statements if one were to look into this seriously.

- Given the importance of Analysis, see the points mentioned in section 2.2 above, about the confusion about limits and the costs for education, both for students and for the curriculum on physics and so on.
- PM. I looked at Hulshof's syllabus in December 2016 and made some comments, and he made some adjustments, but my comments were targeted at that he stopped writing, and I haven't looked at his adjustments.

### 3.2. W.r.t. "Conquest of the Plane" (COTP) (2011)

When I ask you "tell me what you think about my own exposition, in COTP", then I must mention that one member of your Mastermath team, Jeroen Spandaw, has written a "review" in 2012 for *Euclides*, the journal of NVvW. The latter is not a scientific journal. Yet, Spandaw breached scientific integrity. He doesn't present the algebraic approach to the derivative in the book, and he doesn't mention the dynamic quotient, but starts explaining Weierstrasz as if I would not know it. Unfortunately KNAW / LOWI / TU Delft didn't set up a reading committee, and thus they didn't check the breach. Details can be found here.<sup>61</sup>

Professor Hulshof writes me on January 7 2017:

"Prima vent (...). Spandaw ook trouwens, al was die recensie minder gelukkig. Zijn materiaal voor die zij-instroom is prima."

I would like to know what is "less fortunate" about that "review". A breach of scientific integrity should not be called "less fortunate".<sup>62</sup> Hulshof neglects the issue for 4 years now. He claims to care about education but here he can help out with some study and a statement about the mathematical quality of "Conquest of the Plane" and the abuse in this so-called "review", and he neglects it.

There is some overlap in topics, as Spandaw is involved in the Mastermath course on Geometry while "Conquest of the Plane" deals with analytic geometry and calculus.

## 4. On "Foundations"

### 4.1. Logic

The syllabus Edixhoven & Van den Bogaart, "Fundamenten", doesn't refer to my book (1981 unpublished, 2007, 2011) "*A logic of exceptions*" (ALOE),<sup>63</sup> even though there was a favourable review by Richard Gill.<sup>64</sup>

Neither Edixhoven nor Van den Bogaart can claim to be an expert on logic. They would refer to logicians, yet they would be unable to determine what argument would be correct.

Rather than neglect the issue, the proper response would be to identify the difference in opinion. Mathematicians should also protest against breaches in scientific integrity.

The Liar paradox of "This sentence is not true" can be resolved by:

- (1) Russell's theory of types, but this denies useful self-reference
- (2) Brouwer's and Gödel's "proof theory" (replacing truth by "there is a proof"), but this only works for silly systems and collapses again to truth if you require normal connections between proof and truth

<sup>61</sup> <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

<sup>62</sup> <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

<sup>63</sup> <http://thomascool.eu/Papers/ALOE/Index.html>

<sup>64</sup> <http://www.nieuwarchief.nl/serie5/pdf/naw5-2008-09-3-217.pdf>

- (3) three-valued logic, but only for systems that avoid the Liar paradox for three-valued logic (as ALOE shows how this can be done).

It really should not be so difficult to restate these three options and refer to ALOE.

## 4.2. ZFC

The syllabus Edixhoven & Van den Bogaart, "Fundamenten" (2015/12/02), p 55, has this curious statement:

"Voordat we echt aan het werk gaan willen we nog een beschrijving geven van wat wij denken dat wiskundigen moeten doen, in de stijl van de eed van Hippocrates voor medici.

*Wiskundigen geloven in de consistentie van ZFC zolang het tegendeel niet bewezen is. Zij geven stellingen die in de taal van ZFC geformuleerd kunnen worden en zij geven bewijzen van die stellingen die zo begrijpelijk mogelijk zijn voor hun collega's. Op verzoek helpen ze collega's hun werk te begrijpen. Met het oog op de komst van betrouwbare proofcheckers proberen ze bewijzen te geven die met zo min mogelijk moeite uitgewerkt kunnen worden tot formele bewijzen."*

Well, my book (2015), "*Foundations of Mathematics. A Neoclassical Approach to Infinity*" (FMNAI), contains a proof that ZFC is inconsistent.<sup>65</sup>

I am also suggesting to the mathematics community that this proof is subjected to the "automated proof" algorithms that exist today.

A problem here is that I asked Edixhoven a question, and that he didn't reply to this. Well, I am no mathematician in the sense of having a mathematics degree but some might argue that econometrics is applied mathematics. Whatever. I am certainly qualified to ask questions when mathematicians show inconsistencies. It really should not be too difficult for Edixhoven to reply to the question or to check the proof of inconsistency. Yet, Edixhoven neglects the issue, and doesn't mind to present future teachers of mathematics with his delusion that ZFC is consistent though he has been informed that it isn't.<sup>66 67</sup>

## 4.3. Group theory

(1) The "Foundations" syllabus puts emphasis on group theory at the cost of attention for the notation of numbers, while the latter is of key importance for education.<sup>68</sup> See also the discussion of "fractions" below.

(2) In the syllabus Edixhoven & Van den Bogaart, "Fundamenten", examples of the various groups are missing, except for the standard number systems. Given the intention of training for mathematics, the focus is on mathematical abstraction, but teachers are better served by *both* practical relevance *and* mathematical abstraction.

The requirement of more examples is in agreement with Wansink (1971) "*Didactische oriëntatie voor wiskundeleraren*" p141. Teachers better give examples to students that aren't confusing.

"Het is gewenst de eigenschappen van de optelling en van de vermenigvuldiging in het aanvangsonderwijs zodanig te presenteren, dat er een goede grondslag wordt gelegd voor een in de latere fase aan te brengen inzicht in de structuur van groepen.

<sup>65</sup> <http://thomascool.eu/Papers/FMNAI/Index.html>

<sup>66</sup> <http://thomascool.eu/Papers/ALOE/2015-05-21-A-breach-of-integrity-on-paradoxes.pdf>

<sup>67</sup> <http://thomascool.eu/Papers/ALOE/2014-10-29-Cantor-Edixhoven-02.pdf> NB. Reported before FMNAI

<sup>68</sup> <https://boycottholland.wordpress.com/2017/01/03/education-group-theory-and-division>

Het wiskundeprogramma voor onze scholen gewaagt alleen maar van de groep van de congruente transformaties en van de groep van de gelijkvormigheids-transformaties. Maar m.i. zal inzicht in de groepsstructuur eerst dan optimaal tot zijn recht kunnen komen al de leerlingen met een groter aantal verzamelingen kennis maken waarin een groepsstructuur kan worden aangebracht naast andere waarin zulk een structuur ontbreekt."

My own comment is: <sup>69</sup>

#### QUOTE

The change from ring  $Z$  to field  $R$  is not quite the inclusion of division – since the ring already has implied division namely as repeated subtraction – but the change consists of extension of the set of elements by inverse elements  $x^H$  for  $H = -1$  such that the results of division are also included in the same set. In terms of  $Z$  the expression  $2^H$  is not a number, but for  $R$  we accept this.

If the ring has variables and expressions, then we can form the expression  $1 = 2z$ , and we effectively have  $z = 2^H$ , and then we might wonder whether it actually matters much whether this  $z$  belongs to  $Z$  or not.

Thus we get a distinction between:

- Mathematics for which group theory isn't so relevant – such that there is a logical sequence from natural numbers to integers, to rationals, to reals, to multidimensional reals, for, all is implied, and only the end result matters,
- Mathematics for models for which group theory is relevant – i.e. for models for which it is crucial that e.g.  $Z$  has no  $z$  such that  $1 = 2z$ . The crux lies in the elements of the sets, as the operations themselves are actually implied.

#### UNQUOTE

Let me add at this spot that potentially a distinction might exist w.r.t. the algorithms that are allowed per domain (or representation of number).

(3) For Mastermath students, it would also be relevant to point to Richard Skemp (1971, 1975), "*The psychology of learning mathematics*", Pelican, who also discusses the number systems in surprising detail, with the operations like  $+$  dependent upon the kind of set.

Edixhoven & Van den Bogaart likely would claim that they are more rigorous than Skemp, but then it would be helpful to point to Skemp's omissions that might cause disastrous confusion.

And, where does this leave highschool students, who would not benefit from the rigour provided by Skemp, Edixhoven & Van den Bogaart ?

Or, is this exposition in "Fundamenten" and by Skemp needlessly complex, and would it suffice to work within  $R$  throughout ? Thus, teach  $N$ ,  $Z$  and  $Q$  seen from  $R$ , and not the other way around ? A group theorist might remark that a bottom-up way would require symbols  $+_N$ ,  $+_Z$  and  $+_Q$  too, but this is more reason to adopt  $R$  as the general environment. When John from Venice, USA, visits Venice, Italy, and then is called "the American" by his tour guide, then he remains the same person nevertheless, so why would his tour guide not simply call him "John" (presuming that there are no other "John"s around) ?

This approach is clarified on this webpage. <sup>70</sup> I expect mathematicians to recognise that this is mathematically a more elegant solution for education.

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<sup>69</sup> <https://boycottholland.wordpress.com/2016/12/29/what-is-the-use-of-group-theory>

<sup>70</sup> <https://boycottholland.wordpress.com/2017/01/03/education-group-theory-and-division>

(4) In the paper "*Education, division & derivative: Putting a Sky above a Field or a Meadow*" (2014) I have tried to provide group theorists with some suggestions for a theoretical development of the algebraic approach to the derivative, and notably the concept of *dynamic division*. Unfortunately, group theorists may not be looking at these suggestions. It would seem to me that future teachers of mathematics would be aware of these suggestions,

#### 4.4. "Fractions"

For education, the nomenclature of "fractions" is essential. For teachers it is important to have clear notions.

Jan Bergsta and I agree that Jan van de Craats makes a mess of this <sup>71</sup> <sup>72</sup> (though I don't know whether Bergstra and I agree on our critique). My explanation is that Van de Craats has no qualification for education. <sup>73</sup>

Van de Craats collaborates with Rob Bosch, who has a teaching degree, but from a period with little emphasis on didactics. Their "Basisboek" is a repertorium and not a development from didactics. <sup>74</sup>

In the syllabus "Fundamenten", page 4, Edixhoven & Van den Bogaart indicate that  $Q$  is the collection of all "rational fractions", and page 66, theorem V.1.6, gives the arithmetic of "fractions".

- But they don't define what such "fractions" would be. All this "rigour" hangs in the air.
- This doesn't solve basic questions like what is the difference between  $1/2$  and  $2/4$ , and what would be the denominator of  $6$ , when it could also be written as  $36/6$ .

Teachers need such clear terms, provided by fundamental insights in mathematics. The syllabus doesn't provide these essentials for teachers. The syllabus provides a development of "group theory" as the escape route that academics have chosen to avoid such questions, embellished with jargon like "equivalence class" that one cannot use in class.

A step towards resolving such issues was the notion of "proportion space", see "Conquest of the Plane", page 77. <sup>75</sup>

I wrote an exposition to clarify the distinction between fairly useless "group theory" and necessary insight in notations (like in programming too). <sup>76</sup> This discussion avoids the word "fraction" since there is so much confusion about it. I might give a definition, but it is wise to refrain from this, since this might cause needless discussion (or others, like on wikipedia, might use it without reference as if that definition has been in existence since forever).

#### 4.5. From numbers to neighbourhoods

Edixhoven & Van den Bogaart first present the numbers and only then consider neighbourhoods (balls) (p99). If I understand the approach correctly, they presume continuity of  $R$  and only discuss continuity of functions  $f: R \rightarrow R$ . Page 102 first gives continuity and then the notion of limit.

Recently, I developed the suggestion to start with the continuity of  $R$  and base functions upon this. <sup>77</sup> This would make sense for highschool. No doubt, teachers of mathematics must know how research mathematicians have developed a body of theory that avoids such issues, but, this course is not intended to turn the aspiring teachers into research mathematicians.

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<sup>71</sup> <https://staff.fnwi.uva.nl/j.vandecraats/BreukenJvdCdeel1.pdf>

<sup>72</sup> <https://boycottholland.wordpress.com/2017/01/03/education-group-theory-and-division>

<sup>73</sup> <http://thomascool.eu/Papers/Math/2015-09-15-Breach-by-Jan-van-de-Craats-and-Ben-Wilbrink-wrt-scientific-integrity.html>

<sup>74</sup> <http://thomascool.eu/Papers/Math/2016-03-15-Some-experiences-with-HdS-RB-FvdDS-JvdC.html>

<sup>75</sup> <http://thomascool.eu/Papers/COTP/ConquestOfThePlane.pdf>

<sup>76</sup> <https://boycottholland.wordpress.com/2017/01/03/education-group-theory-and-division>

<sup>77</sup> <http://thomascool.eu/Papers/Math/2016-11-30-Algebraic-Approach-Derivative-and-Continuity.pdf>

## 4.6. Complex numbers and linear algebra

As shown in "Conquest of the Plane", I agree that essential definitions of complex plane and linear algebra belong to the foundations for analytic geometry.

## 5. Qualifications of the Mastermath teacher trainers themselves

Let me try to give an overview, alphabetically. See the cv's above.

- A name is put in bold when there is some teaching experience in highschool. All such experience is only short.
- Some are in "teacher training" but presumably not in didactics but in mathematics.
- For some, there are additional comments given my experience since 2008.

When mathematicians get involved with mathematics education, even when they have a few years of teaching experience, then they are at risk of making errors of abstraction that empirical researchers are less likely to make.

Anthonissen, Martijn: at ESoE yet unknown to me how this relates to highschool.

**Bogaart**, Theo van den: taught at highschool for some years and was secretary of cTWO, but see this text.<sup>78</sup>

Bosma, Wieb: apparently no highschool teaching experience. See this letter to Epsilon Uitgaven.<sup>79</sup> Compare with Henk Broer, that involvement with Epsilon Uitgaven doesn't mean that this indicates particular knowledge about education.<sup>80</sup>

Cator, Eric: apparently no highschool teaching experience.

**Daems**, Jeanine: taught highschool in 2010-2013. I am not impressed by the fact that she translated an English book on history into Dutch.<sup>81</sup>

Edixhoven, Bas: see these texts.<sup>82 83</sup>

Hulshof, Joost: apparently no highschool teaching experience. See the **Appendix** below.

Jeurnink, Gerard: was chair of PWN / NOCW and didn't respond to my suggestion that I would present there the new results of "Conquest of the Plane".<sup>84</sup>

Kraaikamp, Cor: apparently no highschool teaching experience.

**Spandaw**, Jeroen: highschool teaching experience, yet, see this text.<sup>85</sup>

Sterk, Hans: at ESoE yet unknown to me how this relates to highschool.

Thije Boonkkamp, Jan ten: likely no experience as highschool teacher.

Top, Jaap: likely no qualification for teaching highschool.

Wepster, Steven: no cv online. As he writes on the history of mathematics (and potential its education), I would like to know his reaction to the diagnosis of the abuse of Van Hiele by Freudenthal.<sup>86</sup>

Wiegerinck, Jan: apparently no highschool teaching experience. See the **Appendix** below.

## 6. Conclusions

The Mastermath programme for "zij-instromers" doesn't seem to fit basic requirements (a) for didactics of mathematics for highschool itself, (b) w.r.t. the background of the "zij-instromers", often in empirics.

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<sup>78</sup> <http://thomascool.eu/Papers/AardigeGetallen/2016-11-02-VandenBogaart-Kool.pdf>

<sup>79</sup> <http://thomascool.eu/Papers/AardigeGetallen/2015-10-30-Brief-aan-redactie-Epsilon-Uitgaven.html>

<sup>80</sup> <http://thomascool.eu/Papers/AardigeGetallen/2016-10-05-Afscheidsrede-Henk-Broer-en-diens-KNAW-sectie-voorzitterschap.pdf>

<sup>81</sup> <http://thomascool.eu/Papers/AardigeGetallen/2016-12-19-Euclides.html>

<sup>82</sup> <http://thomascool.eu/Papers/ALOE/2015-05-21-A-breach-of-integrity-on-paradoxes.pdf>

<sup>83</sup> <http://thomascool.eu/Papers/ALOE/2014-10-29-Cantor-Edixhoven-02.pdf>

<sup>84</sup> <http://thomascool.eu/Papers/AardigeGetallen/2015-10-17-Aan-TK-commissie-OCW.html>

<sup>85</sup> <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

<sup>86</sup> <http://www.wiskundebrief.nl/718.htm#7>



The staff of trainers has inadequate background in didactics (let alone empirics). As they don't know about these matters then it doesn't surprise that these aren't taken care of.

This is no abstract observation but is supported by facts since 2008, see also the reports in the footnotes. Even when a teacher-trainer has some experience as teacher of mathematics, then there are still errors that indicate an abstract mode of thinking, while someone trained for empirics and respect for the facts is likely to react differently.

Part of the evidence consists also of the discussion above of two elements in the programme: "Foundations" and "Analysis".

The discussion also shows how the course programme can be improved.

### **Appendix. Letter to Wiegerinck w.r.t. Mastermath course in Analysis**

(The letter is in Dutch.)

Date: Wed, 28 Dec 2016  
To: j.j.o.wiegerinck  
From: Thomas Cool / Thomas Colignatus  
Subject: Mastermath cursus Analyse 2017 met professor Hulshof  
[Cc: "Bergstra, Jan" & "Hulshof, Joost"]

Geachte professor Wiegerinck,

U geeft de Mastermath cursus Analyse met professor Hulshof, klaarblijkelijk voor zo'n 5 studenten.

Ik heb naar het begin en opzet van de syllabus Analyse 2017 van Hulshof gekeken, en daar enkele opmerkingen over gemaakt. Hulshof heeft aanpassingen gemaakt, en vermeldt dat ook:

<https://www.beteronderwijsnederland.nl/content/cursus-analyse-voorjaar-2017>

Echter, mijn opmerkingen hadden de bedoeling dat Hulshof en u zouden besluiten deze syllabus niet te gebruiken, omdat het op allerlei punten didactisch rammelt.

Mijn advies zou zijn om het recente boek van Michael Range te gebruiken: "What is Calculus?: From Simple Algebra to Deep Analysis".

<http://www.worldscientific.com/worldscibooks/10.1142/9448>

[Addendum 2017-01-08: I will make some more comments on Range's book yet this doesn't change this advice.]

Relevant blijft dan mijn commentaar:

<https://boycottholland.wordpress.com/2016/12/26/descartes-wins-from-fermat-on-the-incline>

Een traditioneel boek is natuurlijk dat van Adams & Essex, maar zie dan mijn commentaar:

<https://boycottholland.wordpress.com/2016/12/08/finally-a-sound-interpretation-for-differentials>

Aldus adviseer ik een reeds bestaand boek te gebruiken, en dit eventueel aan te vullen met leesnotities, zoals vertalingen naar het Nederlands en hoe e.e.a. in de schoolboeken staat.

Van belang zijn denklijk deze opmerkingen, reden waarom ik ook kopie doe aan Jan Bergstra:

(1) Het kan lijken alsof professor Hulshof iets van didactiek weet, omdat hij diverse teksten daarover heeft, bijvoorbeeld op de website van "Beter Onderwijs Nederland" (BON). Maar, deze teksten schrijft hij blijkbaar zonder de vereiste bevoegdheid t.a.v. didactiek. Hulshof verzekert mij dat hij geen uitspraken over didactiek doet, maar alleen als wiskundige commentaar geeft op teksten van didactici. Maar het leest niet zo. Op veel teksten is veel aan te merken. Het betreft ook teksten t.a.v. werk van anderen, reden voor mij om daar niet op te reageren. Hulshof claimt dat veel wiskundigen hem binnenskamers gelijk geven, maar dat is een wonderlijke claim, want waarom zou je dit binnenskamers houden, en hoe moet je zo'n claim controleren (waar gaat het precies over) ?

Als voorbeeld t.a.v. didactiek kunt u kijken naar wat Hulshof zegt over "wiskundige denkactiviteiten" (WDA) en wat ik daarover zeg als bevoegd leraar wiskunde. Ik schreef later dan Hulshof, maar, ik ontdekte zijn tekst pas doordat en nadat ik mijn eigen tekst had geschreven. Niet geheel toevallig kijken wij allebei naar de eerste presentatie door Jos Tolboom, die het voorbeeld geeft van de zichtlijn in relatie tot de straal van de Aarde (zoeken op Tolboom):

Hulshof onder "modelactiviteiten" met de file poppetje.pdf

<https://www.beteronderwijsnederland.nl/content/het-epsilon-handboek-voor-de-didactiek-van-de-wiskunde>

Mijn eigen tekst:

<http://thomascool.eu/Papers/Math/2016-10-31-MTA.pdf>

Wat Hulshof over de wiskunde van dit probleem stelt is grotendeels irrelevant voor de didactiek, en wat relevant is kan een didacticus ook zelf wel verzinnen. Sommige opmerkingen kunnen als didactiek gelezen worden. Met name: "Dat moet natuurlijk precies andersom, eerst de formules uitwerken en daarna de getallen invoeren als dat zin heeft of nodig is." Maar dit is didactisch twijfelachtig. In dit voorbeeld is het niet onredelijk om het probleem niet met algebra te belasten, maar eerst de onbekenden te identificeren en de student de zekerheid geven t.a.v. wat bekend is en wat niet. Zie mijn bespreking, waarin dit beter tot zijn recht komt.

(2) Het is curieus dat Mastermath wil opleiden tot bevoegdheid, maar dit zelf niet eist voor de leraar-trainers. Ik zal hierover nog een aparte brief aan Mastermath sturen. Ja, het gaat Mastermath om cursussen wiskunde, en niet om didactiek van wiskunde. Ik verwijs naar die komende brief.

(3) Mijn ervaring toont dat het bijna onmogelijk is om met professor Hulshof \*\*\* per email \*\*\* te communiceren. Ik heb geen andere ervaring dan email en een enkele lezing door hem. Wanneer ik Hulshof vraag "wat vind je van deze appel" dan antwoordt hij "ik hou van peer". De lezing was voor de NVvW, en in plaats van dat hij zijn gedachten over didactiek presenteerde, gaf hij een triviaal wiskundig probleem waar hij moeilijk over deed. Maar de uitnodiging was juist wegens zijn [onbevoegde] stellingen over didactiek, niet wegens zijn achtergrond in wiskunde.

(4) Afgelopen augustus diende ik een artikel in bij Nieuw Archief over de algebraïsche aanpak van de afgeleide. (Dit staat in verband met bovenstaand boek van Range.) Dit artikel werd met krom commentaar afgewezen, en zowel Hulshof als Bergstra waren zo aardig naar het referee rapport te kijken. Beiden gaven kritiek op dat leesrapport. Hulshof verwees naar een tekst van hem en professor Ronald Meester waarin ondersteuning voor die algebraïsche aanpak zou blijken. Toen ik daarna keek, zag ik dit niet. Hulshof en Meester verwijzen nog steeds naar de benaderingsfout bij de afgeleide, en hebben nog steeds de limiet nodig. Derhalve moest ik de redactie van NAW inlichten dat Hulshof hen en mij op het verkeerde been zette, en dat ik dit commentaar van Hulshof niet kon presenteren als ondersteuning voor de algebraïsche aanpak.

Hier is het artikel:

<http://thomascool.eu/Papers/Math/2016-08-14-An-algebraic-approach-to-the-derivative.pdf>

en hier is de email-geschiedenis:

<http://thomascool.eu/Papers/Math/Derivative/2016-11-21-Breach-integrity-of-science-by-NAW-on-derivative.pdf>

PM. Het betreft hier wetenschappelijke emails en niet persoonlijke zaken, dus na ampel beraad heb ik ze op het web gezet. Ik, Bergstra, Hulshof en redactie NAW zullen hier niet zaken beweren die ik en zij niet tegen anderen staande zullen willen houden. Anderen toegang geven tot deze discussie is mijn enige bescherming tegen de kromme aanpak door de redactie NAW en de verwarring door Hulshof (alsook andere kritiek door Bergstra).

(5) Eerder was Hulshof bestuurslid van BON. In de periode daarvoor hadden enkele wiskundigen (Gerard Verhoef en Mark Peletier) plezier gemaakt over mijn voorstel tot herontwerp (re-engineering) van het onderwijs in wiskunde. Bijvoorbeeld stel ik voor om voor het meten van hoeken uit te gaan van de eenheid van het platte vlak zelf. Dus niet een hoek van 90 graden of  $\pi/2$  radialen maar gewoon een  $1/4$  vlak. Of een kwart slag in het rond. Deze gedachten worden gewaardeerd door Robin Whitty van "Theorem of the Day", zie onderaan deze pagina:

<http://www.theoremoftheday.org/Annex/taumanifesto.html>

PM 1. De wiskundigen die  $\tau = 2\pi$  voorstellen hebben meestal ook weinig zicht op didactiek. Wiskundig bestaat het probleem niet, want  $\pi$  of  $2\pi$  gebruiken is immaterieel. Er zijn alleen overwegingen van didactiek. Dan lijkt  $\tau$  [  $\tau$  ] teveel op het symbool voor straal  $r$ . Dit geeft hopeloze proefwerken. Mijn voorstel is zowel  $\pi$  als  $\text{Archi} = 2\pi$  te gebruiken, met als symbool voor  $\text{Archi}$  de hoofdletter  $\Theta$ .

PM 2. Ook stel ik voor om naast co-ordinaten  $\{x, y\}$  ook de speciale co-ordinaten  $\{X, Y\}$  te hanteren voor de eenheidscirkel (unit radius circle), zodat de functies  $X = \text{xur}[\alpha]$  en  $Y = \text{yur}[\alpha]$  meteen helder zijn, voor hoeken  $\alpha$  gemeten met maat 1. Natuurlijk  $\text{xur}[\alpha] = \cos[\text{Archi } \alpha]$ .

Echter, bij BON maakten Verhoef en Peletier er plezier over, met laster aan mijn adres. Toen ik dit veel later nadien ontdekte, vroeg ik het bestuur en nadien ook Hulshof om dit te corrigeren. Die lasterlijke teksten moeten maar blijven staan, omdat iedereen kan zien hoe men in die jaren zo reageerde, maar er kunnen excuses komen en doorverwijzingen naar correcties e.d. Noch BON noch Hulshof waren bereid om de integriteit van wetenschap te respecteren en te verdedigen. Hier zijn de relevante links:

<http://www.beteronderwijsnederland.nl/node/4346>

<http://thomascool.eu/Papers/Drgtpe/Crisis-2007plus/2012-01-11-FatsoenMoetJeDoen.html>

[typefout]

(6) Ik heb nimmer een tekst gezien van professor Hulshof waarin hij ingaat op een analyse van me. Mocht hij daar uitspraken over doen, dan lijkt het me heel riskant wat hij daarover zou stellen, want ik heb het niet kunnen controleren. Hulshof verzekert mij dat hij geen uitspraken over didactiek doet, maar alleen als wiskundige commentaar geeft op teksten van didactici. Om onduidelijke redenen reageert hij niet op mijn teksten ter verbetering van didactiek. Als de wiskunde goed is, dan zou hij dat toch ook mogen zeggen.

Bijvoorbeeld schreef ikzelf "Conquest of the Plane" (2011), en ook daar zou Mastermath zijn voordeel mee kunnen doen. Maar ik schrijf u niet omdat ik reclame voor eigen werk wil maken.

<http://thomascool.eu/Papers/COTP/Index.html>

Aldus heb ik een kleine "geschiedenis" met Hulshof. Dit is een disclaimer. Mijns inziens kan ik heel goed afstand houden van persoonlijke opinies of gevoelens in deze. Mijn oordeel t.a.v. de syllabus Analyse 2017 is er niet door beïnvloed.

[2017-01-08: Bijvoorbeeld heb ik Hulshof dank gezegd voor zijn opmerking dat een algebraïsche aanpak zonder limieten mogelijk is voor polynomen, want dit zette me uiteindelijk op het spoor van het boek van Michael Range. Ik zou er op den duur wel van gehoord hebben, maar er is nu grote tijdswinst.]

Mijn suggestie is dat u alvast naar het bovenstaande kijkt. Ik kan dit email opnemen als bijlage voor die brief aan Mastermath.

Met vriendelijke groet,

Thomas Cool / Thomas Colignatus  
Econometrist (Groningen 1982) en leraar wiskunde (Leiden 2008)  
Scheveningen