Computer algebra is a revolution. But 21st century skills ? With protest against misrepresentation by Koeno Gravemeijer

Thomas Colignatus
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http://thomascool.eu
Econometrician and teacher of mathematics

Abstract

When you do mathematics on the computer then it is called "computer algebra". Since it is mathematics, it must also be studied in didactics for mathematics. For mathematics education the challenge is to bring computer algebra into the textbooks and the schools in ways that work. Applications of computer algebra to particular fields must be distinguished from those for learning mathematics proper. My three books that qualify on both issues simultaneously are "Voting Theory for Democracy" (2001, 2014), "A Logic of Exceptions" (2007, 2011) and "Conquest of the Plane" (2011), all applications of Mathematica. Instead of a fruitful exchange of ideas and experiences on education and didactics, the decision making discussion is haunted by ghosts from the past. Hans Freudenthal (1905-1990) created "realistic mathematics education" (RME). This RME was not tested in experimental manner but introduced generally in Dutch education. It appears to be a failure, and not a theory but an ideology. The Dutch government has set up additional courses and exams for secondary education to correct for what now has gone lacking in elementary school. In 2014 it appeared that Freudenthal also committed intellectual fraud on RME by appropriating and misrepresenting ideas from Pierre van Hiele (1909-2010). Koeno Gravemeijer (1946) has been promoting RME since around 1980 apparently without real interest in testing it, without discovering this (obvious) fraud, and has since 2008 not explicitly accepted its failure. Since at least 2001 he argues for "21st century skills", and uses the same arguments as for RME. Gravemeijer has written on computer algebra and supervised the Paul Drijvers (2003) thesis: yet, his wrong handling of didactics makes his expertise on didactics of computer algebra questionable too. Gravemeijer's lecture for the 2015 NVVW annual convention of teachers of mathematics in Holland neglected the failure of RME, was scare-mongering about the risks of the 21st century, and disinformative about the really interesting challenges with respect to computer algebra. The current decision making framework puts teachers in a powerless position, and this can be amended by a Simon Stevin Institute (SSI).

Introduction

There is a seminal revolution on computer algebra but the discussion is dragged down into a morass. Distractive is "realistic mathematics education" (RME). This was presented in the 1970s as the answer to the New Math disaster in the 1960s. RME turns out to be a disaster itself too. It appears that Hans Freudenthal (1905-1990) who introduced it also committed fraud, see Colignatus (2014, 2015). Distractive are the "21st century skills" as the present answer to the RME disaster. It appears that "21st century skills" that apply to mathematics are RME in disguise. What is happening here ? What gives a rational framework to handle the confusions ?

In Holland, a key supporter of both RME and "21st century skills" is Koeno Gravemeijer (born 1946, this year 69). There is no particular reason to single out Gravemeijer except for his speech to the 2015 NVVW annual convention of the Dutch association of teachers of mathematics that I attended. [1]

The following reviews the general argument and is also my protest against the abuse of science and logic by Gravemeijer. This memo provides a rational framework and deconstructs fallacies by RME and "21st century skills", for example on the need of teaching arithmetic that fits the so-called "21st century skills". Other texts by Gravemeijer are from 2002, 2013, 2014.

The **Appendix** deconstructs Gravemeijer (2014) by paragraph: Dutch orginal on the left, comments in English on the right. This tabular format allows to see the fallacies, rhetorical techniques (like the straw-man) and disrespect for the intelligent reader.

We first need to develop basic notions before we can do the discussion.

Basic notions

Seminal revolution

We are living in a period with a **seminal revolution** similar to the invention of the wheel, the alphabet and positional number system:

> we can do mathematics on the computer – and it is called "computer algebra".

We know this since 1963 and Project MAC. [2] This is doing mathematics, rather than mere programming or punching buttons. By comparison:

- The arrival of calculators is not much different from the invention of ruler and compass, or the later arrival of tables for trigonometry and logarithms (recovered exponent, rex rather than log). Those are techniques, with the didactic balance of drilling and understanding.
- Doing mathematics on the computer is a game changer.

In this, there is nothing special about the calendar, the year 2000 and the 21st century.

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[1] NVVW website in Dutch, summary of the Gravemeijer 2015 speech: "Globalisering, digitalisering en automatisering komen steeds nadrukkelijker in het nieuws. De maatschappij verandert snel en daarmee ook, wat er nodig is om succesvol mee te kunnen doen. Het onderwijs zal daarop moeten worden aangepast. Het is daarom de hoogste tijd voor een bezinning op het wiskundeonderwijs. Enerzijds omdat wiskunde een steeds grotere rol speelt in onze maatschappij. Anderzijds, omdat steeds meer reken- en wiskundige bewerkingen door apparaten (kunnen) worden uitgevoerd. Dit vraagt om een heroverweging van de doelen van het wiskundeonderwijs. Bovendien zal moeten worden nagedacht over wat digitalisering betekent voor de opzet en uitvoering van het wiskundeonderwijs."

[2] https://en.wikipedia.org/wiki/MIT_Computer_Science_and_Artificial_Intelligence_Laboratory#Project_MAC

Computer algebra became mature in the 1980s. For mathematics education the challenge is to bring computer algebra into textbooks and schools in ways that work. Applications of computer algebra to particular fields must be distinguished from those for learning mathematics proper.

Bottlenecks in mathematics education

The real problem is this:

> **Proposition 1:** Mathematicians are trained for abstraction. When they come into the classroom, then they see real life students. They resolve cognitive dissonance by sticking to traditional views. Those views are not designed for didactics. Mathematical formats even appear to be crooked, and mathematicians are trained not to see this.
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> **Proposition 2:** Mathematicians tended to despise computer algebra in 1980-2015, even though it was highly relevant for education. Nowadays there is more acceptance, but not necessarily for education. There is little need for teachers and educators of mathematics to use computer algebra, hence see its value.

In itself it is surprising that computer algebra isn't used so much yet. One supposes that the wheel or the alphabet also had to compete with alternatives. Once it is used, it becomes difficult to imagine how people could have lived without it. The market share for the use of computer algebra seems to have stopped at early adopters. There is however an explanation for the current stagnation.

A failed revolution since the 1970s: realistic mathematics education (RME)

In 2015, Holland is trying to recover from "realistic mathematics education" (RME). Correlation is no causation, and there willl be other factors at work, but there still is a causal connection:

- In the period 1970-2015, RME became dominant, with 75% coverage in 1994 and a peak of 100% in 2002-2010.
- In 2015, the government requires separate courses and tests on arithmetic for students in highschool and vocational schools, since they don't adequately learn this in elementary school anymore. [3]

Kees van Putten (2008) answers Adri Treffers, in the latter's denial on the worsening outcomes since 1987 in educational results on arithmetic. The thesis by psychometrician Hickendorff (2011) suggests that RME and "traditional arithmetic teaching" are "equally good" but this research suffers from invalidity since it neglects that arithmetic is relevant for later algebra, see Colignatus (2015). (A quick test is that the words "algebra" and "algebraic" do not occur in the Hickendorff thesis.)

> **Proposition 3:** RME was created by abstract thinking mathematician Hans Freudenthal (1905-1990). Apparently the New Math and behavioral psychology (drilling pigeons) were a disaster, so that Freudenthal got a platform. But RME still is ideology and not empirical science.
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> **Proposition 4:** Freudenthal also committed intellectual fraud by taking ideas from practical teacher Pierre van Hiele (1909-2010), distorting them and presenting those as his own (while the distortion doesn't reduce the theft). See Colignatus (2014, 2015). [4]

In Utrecht there is the "Freudenthal Institute". This term suggests scientific neutrality. This is not deserved. It is better to speak about "Freudenthal Head in the Clouds Realistic Mathematics Institute" (FHCRMI). This denomination is no demonisation but an invitation to study the evidence in the given references.

[3] http://www.wiskundebrief.nl/721.htm#4
[4] http://arxiv.org/abs/1408.1930

Seeming revolution: 21st century skills

Some educationalists, also on mathematics, speak about "21st century skills", with notions like: communication, collaboration, "ict literacy", creativity, critical thinking, problem solving abilities, social and cultural values. [5] These are mostly goals of teaching since antiquity (except that parents may buy education for their children to give them a competitive edge over others), and there is nothing special about the 21st century, except the onset of computer algebra after 1963 that undeniably also continues in the present century. The phrase of "ict literacy" distracts from the real issues (see the Propositions).

If "21st century skills" *on mathematics* merely meant the decent introduction of computer algebra into education, including adaptive testing and assessment, then the discussion would be different. Instead we find a whole range of topics that are rather distractive. Someone is trying to set the agenda, and this someone is not necessarily the teaching community. It is not clear whether this "21st century skills" platform has an origin in industry or that it morphs various educational philosophies like RME.

FHCRMI has also been involved with "21st century skills" with special attention to mathematics. [6] There are now texts under the label of "21st century skills", also presented at the OECD, that are quite similar to RME. You may understand the feeling: plugging the hole in front of you, then another pops up behind you, with the same freezing water.

The movement for "21st century skills" is more diverse than only mathematics but doesn't seem to be properly critical about (i) abstract thinking mathematicians, (ii) RME, (iii) itself. Both RME and "21st century skills" are highly ideological and neglect that didactics is an empirical science – in this case of mathematics.

Abuse of fancy phrases

FHCRMI has a tradition of coining terms, to distract from already known concepts and to suggest something new. It comes with the advantages that you can hide that you have no new insights yourself, and that you do not have to refer to others who already have shown that you were incompetent in the first place.

- Hans Freudenthal coined words like "guided reinvention" and "anti-didactical inversion" so that he didn't need to refer to Van Hiele (1909-2010).
- "Literacy" is a term from the education in language, and the term has been applied by Jan de Lange of FHCRMI to Mathematics, and it has been adopted by OECD PISA. [7] Now there is "ict literacy" as if the notions would be so new.
- Another example is the phrase "think activities". Activities are related to drilling and thinking tends to require that you sit down (Kahneman (2011), *Thinking, Fast and Slow*). Didn't Van Hiele in his 1957 thesis not already discuss the notion of insight ?
- Gravemeijer introduces the phrase "global arithmetic".

Gravemeijer on the combination of RME and 21st century skills

Holland is recovering from the RME disaster. Gravemeijer has criticised this Dutch discussion on recovering from RME as being too much focused on arithmetic and neglecting the preparation of students for the modern information society. [8] This turns the true situation

[5] http://benwilbrink.nl/literature/21st_century_skills.htm
[6] http://www.fisme.science.uu.nl/wiki/index.php/21ste_eeuwse_vaardigheden and http://www.fisme.science.uu.nl/wiki/index.php/21ste_eeuwse_vaardigheden_en_WDA%27s
[7] http://www.oecd-ilibrary.org/education/pisa-2012-assessment-and-analytical-framework_9789264190511-en
[8] http://www.fisme.science.uu.nl/wiki/index.php/21ste_eeuwse_vaardigheden "Koeno Gravemeijer stelt in zijn lezing op 7 maart 2014 (Nationale Rekencoördinator Dag) dat het vreemd is dat in alle onderwijsvernieuwing rondom de leerlijnen rekenen een oriëntatie op de

upside down. It neglects that it have been RME and he himself who created chaos. There still is no stated admission by Gravemeijer that RME is a failure, and explanation why. This denial cause that other people lose time now only to restore mathematical competences known since antiquity. A good basis in mathematics is required to deal with computer algebra. It is not sufficient to just have computer algebra: students need knowledge, skills and attitudes also with pen and paper. There are also developments w.r.t. computer algebra that Gravemeijer in this statement neglects, like my three books for mathematics education that use computer algebra.

My background

I myself have been a user of the computer algebra program *Mathematica* (by Wolfram Research Inc.) since 1993. I stated already at that time that computer algebra is like the invention of the wheel and the alphabet. I sell *"The Economics Pack. Applications of Mathematica",* my collection of applications. My book *"Elegance with Substance"* (2009, 2015) contains a discussion from 1999, *"Beating the software jungle",* with arguments that are repeated here. My three books are *"Voting Theory for Democracy"* (2001, 2014), *"A Logic of Exceptions"* (2007, 2011) and *"Conquest of the Plane"* (2011), all applications of Mathematica. These are applications to fields but also generate mathematical understanding.

Thus I write also from own experience – which the reader may see as a disclaimer too. Propositions 1 & 2 are based also upon observation as an eye witness. As far as I know, FHCRMI including Gravemeijer haven't shown an interest in my books that use computer algebra.

Governance of mathematics education and the Simon Stevin Institute (SSI)

Thus, what Holland tries to repair on RME may be re-introduced under international pressure before teachers get involved. It is crucially important to be aware of the power unbalance w.r.t. education and didactics. While the demand side is organised – governments are in the driver seats as to what should be taught - there is institutional chaos on the supply side. A key problem is that teachers of mathematics have no proper platform to discuss issues in scientific manner, with this harrassment by ideologues and non-empirical mathematicians. [9]

> **Proposition 5:** The evidence about the failure of RME is also evidence of the disastrous impact of the lack of *organised* influence by teachers on mathematics education. This failure of RME warns about the prospect for "21st century skills", and in Holland the Onderwijs2032 discussion. There is need for a Simon Stevin Institute (SSI). [10]

The Dutch Council on Education Onderwijsraad (2014) also speaks about "21st century skills" and concludes to the need of a new national authority to set the curriculum. Apparently the curriculum must be set by specialists, who need not be teachers. My suggestion instead is to have this SSI, such that teachers do also scientific research, and have their say about educational values, curriculum and didactics. Currently, it is a problem indeed that most teachers know little about computer algebra: since they don't use it. The focus of teaching is confined to given curricula and the classroom, while my proposal is to have Academic Schools that allow teachers to do research on education, the curriculum and didactics.

In Holland, the Inspection is the guardian appointed by the government to check whether schools do what they are supposed to do (and for which they receive funds). Currently the Inspection ressorts under the government, but when the power unbalance in mathematics education is resolved, it would ressort under the national institute for mathematics education - proposed name: Simon Stevin Institute (SSI) - since the key control parameters are

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reken-wiskundige kennis die mensen nodig hebben om in de informatiemaatschappij te functioneren, ontbreekt."

[9] https://boycottholland.wordpress.com/2015/10/31/the-power-void-in-mathematics-education

[10] http://thomascool.eu/Papers/AardigeGetallen/2015-10-17-Aan-TK-commissie-OCW.html

educational values, curriculum and didactics. [11] Obviously, this SSI would be based in the empirical science of educational research, and be responsive to practical teachers rather than ideologues.

As an example of the current power unbalance, consider the report by the Dutch Inspector of Education Onderwijsinspectie (2002), on ICT and education on mathematics and arithmetic. It is a horror show. I will say a bit more on this below. The Inspector of Education seems to rely on FHCRMI and its associates. FHCRMI has a tradition to program in single-purpose menu-driven push-button java-applications, instead of using an integrated computer algebra package that allows applications to build upon each other, and that allows teachers and pupils to further develop and adapt for suitable purposes.

In the current situation of unbalance we must perhaps wait for the application that teachers can use professionally – adaptive testing and assessment (Maple TA) – after which they can understand more about the seminal revolution that is taking place w.r.t. doing mathematics on the computer. But it also appears in this memo that policy makers on education do not get the proper information from said blind mathematicians and related educators (like Gravemeijer).

Structure of the argument by Gravemeijer (2014) on 21st century skills

Koeno Gravemeijer holds with hardly any evidence:

(1) that there *are* 21st century skills, also indicated by economists (though he is no economist and doesn't refer to views by critical economists)
(2) that the arrival of calculators and computers require a (vague) change in the teaching of mathematics (Sputnik 1957, computers were already a hot item in 1963, [12] Microsoft founded in 1975, Wolfram Research Inc. founded in 1987 for *Mathematica,* a system for doing mathematics on a computer [13])
(3) that calculators and computers are even more important after 2000 since there is now talk about "21st century skills" (as if the calendar matters) and this provides a welcome bandwagon to save RME
(4) that the solution is "realistic mathematics education" (RME), originally presented somewhat later than 1957 for somewhat other reasons than calculators or computers: but computers and such "21st century skills" can be usefully included in the arguments for RME, even though RME has shown to be disastrous
(5) that it is possible to *say* that some things in education must change (by implication also in RME) and to *hold* at the same time that RME should not be adapted, which is a remarkable agility with dealing with veracity.

I will reply by giving evidence, both as an econometrician and teacher of mathematics. Note that I am qualified for teaching at secondary and tertiary education but not at primary education, while a key question in this discussion is whether (small) children should learn counting and arithmetic with their fingers or on the computer (tablet). My suggestion is an enquiry by parliament, that had authority and ample funds and can call in the help of other scientists versed in experimental designs involving children.

Neoclassical view on economics and mathematical skills

The argument about disappearing jobs due to technology and international competition with low wage countries (also low wage engineers) is scare-mongering, since technology and trade are sources of welfare. The real issue is how goverments are distorting markets with regulations and taxes. Fellow-economists writing on the impact of trade and technology (not only computers) better stop writing and first read my books (PDFs online):

[11] https://boycottholland.wordpress.com/2015/10/31/the-power-void-in-mathematics-education
[12] https://www.youtube.com/watch?v=Q07PhW5sCEk
[13] See various computer algebra systems: https://en.wikipedia.org/wiki/Computer_algebra_system

(i) *"Definition & Reality in the General Theory of Political Economy"* (DRGTPE) on economics [14] and
(ii) *"Elegance with Substance"* (EWS) on mathematics education and the political economy of the mathematics industry.

DRGTPE presents new insights in economics. EWS has new insights on its subject too. These books present the evidence, and there is no need to further discuss this here. For Holland I propose two parliamentary enquires to study on the evidence and draw policy conclusions.

Gravemeijer neglects these books though EWS was reviewed in 2010 in *Euclides,* the Dutch journal for teachers of mathematics. Thus in the small research community of Holland, Gravemeijer writes about the economic impact and supposed need for educational changes, but neglects a different view close at hand without dealing with the arguments. (Ben Wilbrink lists various sources and other criticisms. [15])

On Gravemeijer (1994) and Van Hiele's theory of levels of insight

When a person is affiliated with the Freudenthal Head in the Clouds Realistic Mathematics Institute then it is advisable to check how he or she writes about Van Hiele's work. The Gravemeijer (1994) thesis is online. [16] P22-23 correctly summarizes Van Hiele's level theory. He also correctly quotes Van Hiele (1973) *"Begrip en inzicht"* p182-183.

> "Whereas at ground level the concept 'four' may be tied to visible entities, e.g. to the vertices of a square, and features as a word in the series 'one, two, three, four, five, ...', on the first level it is a junction in a relational framework. On this level it might be two plus two, or two times two, or possibly five minus one. In any case it has already disengaged itself from the realm of the concrete." (Van Hiele p182 quoted by Gravemeijer 1994:23)

Gravemeijer p23 concludes fairly:

> "For the authors of R&W [a textbook developed by him and others], the significance of the level theory did not reside in its theoretical use, for example a sharp classification in levels, but in its practical implications. First, mathematics has to start on a level at which the concepts used have a high degree of familiarity for the students, and, secondly, its aim has to be the recreation of a relational framework. The selection of Van Hiele's level theory also had consequences for the curriculum goals: rather than aiming for isolated skills or basic facts, courses would be aimed at the creation of relational frameworks. In more concrete terms, numbers up to 20 would eventually function as junctions in a relational framework."

In the **Appendix** we will see Gravemeijer consistently argue for a "network", required for the development of proper number sense and algebraic sense. However, the errors are:

(i) allowing pupils to rediscover relations: allowing them to get lost or take too much time
(ii) overindulging: too many exercises, again and again requiring the same discovery.

This overindulging is an abuse of the work of Van Hiele, since these two errors cannot be logically tried to this work. Reading the work of Van Hiele one gets the impression that he is rather traditional in terms of guiding pupils along the path of the traditional algorithms like long division. Thus, while it is important to develop relations, it is important to see that the tables of addition and multiplication already provide such relations, and that awareness of those can be generated by discussing and memorising and gradual rising experience. Yes, one can bring a horse to the water but not make it drink. The pupils should have freedom for their own creativity so that the penny can drop. But the errors above are in the principles that cause

[14] Dutch readers can benefit from D&S: http://thomascool.eu/SvHG/DenS/Index.html
[15] http://benwilbrink.nl/literature/21st_century_skills.htm
[16] http://repository.tue.nl/443094

excess – caused by Freudenthal's misrepresentation of Van Hiele, and duly copied by Gravemeijer.

Gravemeijer (1994:25-26) takes only three levels. With the third level given as group theory and its feature to enter into the didactics of proof, he is forced to assign the associative, commutative and distributive rules of arithmetic to the first level of relations (above the basic level). It is more useful to have four levels, with those rules as a separate intermediate level. In that case the shift from numbers to variables is gradual and natural, since variables are handled more via rules than numbers. Van Hiele (1973:199-200) proposes the introduction of the Abelian groups for addition and multiplication early in education, since it is easier to discuss the notion of proof with arithmetic than with geometry. It is not clear to me whether Gravemeijer discussed this. It does seem that the years 1957-2015+ have been lost for didactic improvement according to Van Hiele.

On Gravemeijer (1994) and RME

This present deconstruction of Gravemeijer and "21st century skills" somewhat overwhelms me. At first I thought that the deconstruction in the **Appendix** should be sufficient. But Gravemeijer's argument on "networks of relations" reminded of Van Hiele and caused me to look at his 1994 thesis, see the discussion above. For the rest I looked at it only diagonally. Perhaps I should look into it deeper – but there is also lack of time and urgency. For this memo my position is: (i) RME is bankrupt, given the Dutch evidence, (ii) it is fair to take some points from the 1994 thesis to give some indicative links for who wants to delve deeper.

Gravemeijer (1994) Section 6.1 on "evaluation research" gives his view on empirical tests on RME. He distinguishes curricula (his topic) from other issues (practical teaching). A statement:

> "My suspicions are that the realistic curricula in The Netherlands will surpass their competitors in the area of learning results."

There is a reference to Wijnstra 1988 (PPON by CITO), and he calls it "quite convincing" (with only reference on p136 to Treffers 1988 that the distinction would be positive for RME). He might have regarded CITO as better equiped rather than himself to do the actual testing.

When we observe in 2015 that RME doesn't work, Gravemeijer (1994) Section 6 also indicates the true RME way out. RME might not be properly executed. It might be that teachers use RME books but continue teaching in traditional manner, for example pick out only the sums and start drilling again. He sees two possible remedies: either fully work out proper RME didactics and put this in the textbook fully, so that the teacher may also be an actor playing a script, or resort to indoctrination, such that teachers know RME by heart and will not deviate from the true gospel. Gravemeijer (1994:175):

> "For the time being, two paths are available for improving the implementation:
> - directly influence the teachers' views, knowledge, insight and skills (...),
> - choose a more directed form of realistic mathematics education and adapt the textbooks accordingly."

It remains remarkable that the option that it doesn't work isn't mentioned. This however can be explained by the expectation that the method will be successful, which expectation is so great that the method was introduced without proper testing on lab rats.

The degree of control that Gravemeijer specifies is proper for a controlled experiment, and generally inadequate for a field test. When he requires such a degree of control, why didn't he design such a controlled experiment ?

Gravemeijer's lack of teaching practice and empirical testing

The 1994 thesis by Koeno Gravemeijer's informs us that he first studied mathematics and physics, majoring in nuclear physics, in Amsterdam, and subsequently education, majoring in structural design, in Leiden. His curriculum vitae does not mention practical experience in teaching mathematics at any level (primary, secondary, tertiary). There is no stated evidence of having been involved systematically in proper empirics, e.g. empirical modeling, testing or experimental design. The thesis mentions an "experiment" on a number line, a test at an American school, and some schools are called "experimental schools". Chapter 5 contains a few statistics on curricula but I have not looked at it to see what it means and whether it is relevant or valid, either in 1994 or 2015. I haven't looked at his list of publications.

Having listened to his 2015 NVVW lecture, my impression is that Gravemeijer is more an abstract thinking mathematician than an empirical researcher.

He worked since 1986 at Freudenthal Head in the Clouds Realistic Mathematics Institute in Utrecht en retired as professor in Eindhoven and Utrecht. [17] He has been long involved [18] in RME, say with the MORE (1993) abuse of the work by Pierre van Hiele, [19] and this 1994 thesis is under supervision of Adri Treffers, another pillar of RME. [20]

Rather than admitting failure on RME (the evidence mentioned above), Gravemeijer points to this "21st century skills" discussion, perhaps in real concern about the 21st century, but just as likely to save RME.

On Gravemeijer and didactics of computer algebra

Gravemeijer has written on computer algebra and supervised the Paul Drijvers (2003) thesis, alongside other superviser Jan de Lange. [21] Yet, Gravemeijer has shown a wrong handling of didactics with an emphasis on RME ideology and less interest in empirical mathematics research: thus his expertise on didactics of computer algebra becomes questionable too. With emphasis: *questionable*. This is a new field and all researchers are handicapped. Interactivity with a computer reminds a bit of private tutoring, but the computer can be stupid on questions and fast on complex results; and so on. Doing mathematics on the computer is a game changer. It might be that one must first learn mathematics in the traditional manner before using the computer. Thus there is every reason to be careful.

I was involved in college education 1997-2001 and highschool after 2007 (with a first degree in 2008). There was no cause for me to look at Gravemeijer or Drijvers (2003) on computer algebra, see my objectives in EWS (2009, 2015).

- I do protest that Drijvers in 2012 as editor of the "Handboek Wiskundedidactiek" allowed Gerrit Roorda to be silent on my suggested algebraic approach to the derivative. This

[17] https://www.tue.nl/en/university/about-the-university/eindhoven-school-of-education/about-esoe/staff/detail/ep/e/d/ep-uid/20072724
[18] http://benwilbrink.nl/projecten/positionering.htm
[19] http://thomascool.eu/Papers/Math/2015-09-15-Breach-by-Jan-van-de-Craats-and-Ben-Wilbrink-wrt-scientific-integrity.html
[20] Treffers's thesis "Wiskobas doelgericht" is from 1978, when supervisor Freudenthal (1905-1990) was 73 years of age. Second supervisor was Jelle Sixma (1918-2010) an educator known for his work on reading conditions ("Leesvoorwaarden", 1973). I haven't checked this thesis. One can only hope that Sixma wasn't bullied by Freudenthal into believing that the educational ideas on mathematics were correct. Incidently, there are stages of reading, that remind of the levels of Pierre van Hiele, see https://en.wikipedia.org/wiki/Jeanne_Chall And apparently there are stages in language acquisition https://en.wikipedia.org/wiki/Roger_Brown_%28psychologist%29
[21] http://jdlange.nl/about (thesis 1987 with supervisors F. van der Blij en A. Treffers) http://www.fisme.science.uu.nl/wiki/index.php/Mathematics,_Insight_and_Meaning

wasn't resolved in 2012 and hence I also protest since 2014 that he was appointed professor in mathematics education research in 2014. [22]

- Given the 2014 discovery of Freudenthal's fraud on RME, that is obvious when one starts studying the works by Freudenthal and Van Hiele, it is curious that Drijvers didn't discover this himself, and hasn't responded yet.

- Given Drijvers's stated academic interest it is curious that he hasn't looked yet at my books that are written in the environment of Mathematica (dates 2001, 2007, 2011), and hasn't even stated why he has shown no interest (whether it is because of RME ideology or other). He might also have played a positive role like Christiaan Boudry w.r.t. an improper "review" but apparently did not. [23]

The mentioning of these various names should not distract. My proposal has been an enquiry by Dutch parliament and the creation of a Simon Stevin Institute that would create an environment to discuss such issues in proper scientific fashion.

A problematic text by the Dutch Inspection of Education 2002

Looking into this subject, Google generates also this report by Onderwijsinspectie (2002), that explicitly deals with ICT (information and communication technology) and "arithmethic and mathematics for the 21st century", in which they refer on p4 to Gravemeijer (2001, 2002).

There are various institutional connections and flows of funds. Getting rid of the RME and "21st century skills" confusions is one thing, but these institutions must also appear to be willing to blink.

There has been a huge waste of public funds, with the finance of all small applets and other computer projects, instead of adopting a fully integrated computer algebra system (that doesn't deal with gadgets but concentrates on what matters: doing mathematics on the computer).

Heck et al. (2008) modestly state: "The main drawback of the generated Java applets is that there is no real computer algebra system behind it yet." A current remedy is to use the Java applets as front-ends, and create a link to the CA system in the back, but, why not use CA directly ?

Disclaimer on Mathematica

In the USA there is a general reliance on private enterprise and distrust of big government. In Europe there may still be preference for government intervention under democratic control. Mail delivery is an old government licence, and paradoxically U.S. Mail still exists because it is mentioned in the US Constitution while Holland has now privatised mail delivery.

As a scientist I use a computer with an operating system that are both produced by private companies. Thus I am also reasonable at ease with WRI as a technology firm that develops *Mathematica.* Still, mathematics should be free for all, and there are awkward issues when part of the mathematical language would be claimed as format in a particular computer application. For example, *Mathematica* uses = for Set, == for Equal and === Identical, while Algol (Edzger Dijkstra) used := for Set. Mathematics is also communication, also with a computer, and one must make choices. One can imagine that artists may have some claim on some form that they invent, but for mathematics such notions arise in the literature and it shouldn't be that a technology firm actually uses the need for a convention to create a platform that subsequently is claimed to be their property. Creativity and endurance should meet with rewards but not block such efforts by others.

[22] http://thomascool.eu/Papers/BHRM/Index.html
[23] http://thomascool.eu/Papers/COTP/LOWI/Index.html

Elegance with Substance (2009, 2015) discuss the issue on beating the software jungle. A new analogy is the business model by CITO, the assessment company that derives from psychologist A.D. de Groot. They have both a not-for-profit foundation and a for-profit company. CITO does testing for the government, say all kids graduating from elementary school, which can be seen as a public service which also requires involvement and open access for science. Apparently there are gate-keepers who guard the exchange of R&D knowledge between the two legs of CITO. [24] My suggestion to WRI is to look into this model. When all computer algebra systems can use the same language, then programs can be exchanged, and then competition shifts to relevant areas as it properly should.

Admittedly, the phrase "language" may be too simple. This isn't just the use of the alphabet. Communication between people is not just by talking and (sometimes) listening, but also uses gestures, (motion) pictures, and so on. For computers there is the interface – in *Mathematica* called the Front End. This uses menu's and conventions on what to project on the screen. Apparently there is a growing legal body on the "look & feel" of computer programs. However, one can imagine that education should be able to specify requirements, and that those would create a platform for competition.

Consider the role of Microsoft Word. Admittedly, dedication generates stability. (But it was integrated with Excel into Microsoft Office.) Still, if Microsoft had made Word a public domain program, then it could have been the basis of PDF, e-readers and browsers too, and there would have been less need for other dedicates. An environment for doing mathematics on the computer also requires an environment for text editing, if only to type in answers for an assessment, but also to write books. Currently programmers are forced to recreate the same functionality of Word, and take the advantage of giving their own formats a commercially exploitable "look & feel". This is what I call the "software jungle". Instead, with a common foundation competition is not between English and French, Apple or Microsoft, but on extras and the "je ne sais quoi" that generate productivity growth. Potentially governments have a vested interest for education to create such a foundation: but now crucially with the feature of doing mathematics on the computer.

Conclusions

Our conclusions are:

(1) Once Freudenthal (1905-1990) as a mathematician accepted the 1957 thesis by Pierre van Hiele (1909-2010) on didactics of mathematics, Van Hiele should have become professor in mathematics education research, and Freudenthal should have stopped peddling his educational views unless following proper methodology of science. Freudenthal was already deep into fraud, see Colignatus (2014, 2014), when he promoted Treffers in 1978, who again promoted Gravemeijer in 1994, who promoted Drijvers in 2003. It is absurd that Treffers, Gravemeijer and Drijvers studied the works by Van Hiele and Freudenthal and did not discover Freudenthal's fraud. It doesn't seem likely that the thesis and professorships by Gravemeijer derive from real science.

(2) Gravemeijer should give a clear explanation of the dismal results of RME in Holland, and not dodge the question. Given his stated expectation in 1994 of a success, it is strange that he is not curious about the real outcomes. The more he dodges the issue, see Van Putten (2008) in answer to Treffers, the more he appears to be an ideologue. It becomes ever more likely that he wasn't interested in real outcomes in the first place. It may also be difficult for him to judge on this, because he lacks qualification and actual practice in teaching mathematics at elementary or secondary level (and only non-mathematics at tertiary level).

(3) Gravemeijer (2015) 's presentation at the 2015 NVVW annual convention was a repeat of earlier misconceptions and misrepresentations. Apparently he regards math teachers as people who can be told such stories. Personally I was a bit amazed about the more than polite applause but I also suppose that teachers of mathematics tend to be lacking in

[24] http://www.cito.nl

knowledge of economics. (See EWS w.r.t. some confusions on economics in textbooks on mathematics. Please remember that I am critical of mathematics education.)

(4) Key challenges for mathematics education that don't depend upon the calendar are discussed in EWS (2009, 2015), and neglected by Gravemeijer. Let he explain this neglect. Holland is a small country and foreigners would tend to suppose that locals communicate. Since I listened to Gravemeijer's lecture and made this deconstruction, let he look at EWS.

(5) This discussion is not about ideology but about scientific standards. Naturally there are many other challenges but a core issue is the resolution of the power unbalance in mathematics education, i.e. for Holland the need to create a Simon Stevin Institute. [25] Without such a national body that provides a foundation for this kind of discussion within the empirical science of education research, and that links theory and empirical foundings to educational practice, this present discussion is rather hopeless because quickly soured by ideology, as it apparently has been since Sputnik 1957 and the New Math in the 1960s.

[25] https://boycottholland.wordpress.com/2015/10/31/the-power-void-in-mathematics-education

Appendix: Deconstruction of Gravemeijer (2014)

Tijd voor ander rekenonderwijs	***Comments on this article***
Dutch text by Koeno Gravemeijer (around November 2014 ?) http://www.didactiefonline.nl/blog-blonz/12171-tijd-voor-ander-rekenonderwijs and also http://www.rekenenwiskunde21.nl	English Comments by Thomas Colignatus, December 5 2015 http://thomascool.eu and https://boycottholland.wordpress.com/2015/10/20/hans-freudenthal-petty-crook-no-demon-1
Vrijwel alle rekenbewerkingen die kinderen nu leren, worden in de wereld buiten de school door computers uitgevoerd. Maar dat maakt het rekenonderwijs niet overbodig. Ook om te kunnen werken met apparaten die allerhande rekenwerk uitvoeren, heb je rekenvaardigheden nodig.	The reference to computers is a bit silly. The TI-83 was "advanced" when it was introduced in 1996, see http://mic.com/articles/125829/your-old-texas-instruments-graphing-calculator-still-costs-a-fortune-heres-why (reference thanks to Raymond Johnson)
Maar dat zijn wel andere rekenvaardigheden dan die waar rekenonderwijs zich nu op richt.	Current education has also been targetted at handling the calculator in a decent fashion. One can agree that this education must be changed, but not because of the argument that computers are a novel phenomenon.
Het gaat dan om het kunnen toepassen van rekenkennis, het begrijpen van wat de computer doet en de computer globaal kunnen controleren. Naast meer aandacht voor toepassen en begrijpen, vraagt dit ook om een verandering van leerstof.	• Current mathematics education is already quite focused on applications – see the use of contexts. • Van Hiele indicates that learning goes from concrete to abstract, and Freudenthal misread this as applied mathematics. • The new phrase "global control" for computer use just rephrases what is known since the advent of computers.
Samenleving Computers veranderen de maatschappij op twee manieren, enerzijds door arbeid overbodig te maken, anderzijds door nieuwe werkgelegenheid te creëren. Zo nemen computers allerlei taken over; met name in industriële processen, maar ook door de prijs te berekenen van de groente die je in de supermarkt afweegt, of door bankemployés overbodig te maken bij het opnemen van geld.	• It is well known that computers change society. Microsoft was founded in 1975. • Giving such examples is a bit silly. • Challenges for the future are a bit different than this early phase of computerisation. Challenges to the legal system are for example: privacy and other protection against abuse.
Maar de computer creëert ook nieuwe mogelijkheden, zoals het gebruik van 3D-printers, het analyseren van 'big data' en het doorrekenen van simulaties. In het eerste geval is de computer een concurrent, in het tweede geval een stuk gereedschap dat menselijk handelen aanvult.	• It is again silly to argue that the computer is a tool. It isn't intelligent yet, is it ? • Wolfram Research was founded in 1987. Perhaps G adapted this text a bit from a text from 1990 ?
Vertaald naar het onderwijs betekent dit dat we ons niet zozeer moeten richten op vaardigheden die de computer overneemt.	This can only be a deliberate confusion and hence a fallacy: (a) Nobody claims that people should be trained to beat computers (Kasparov vs Deep Blue 1996). (b) People still need education on mathematics and so on.
Belangrijker zijn vaardigheden die je nodig hebt als je met computers of gecomputeriseerde	Yes, people need an education in mathematics and an education in how to deal with

apparaten werkt of die in bredere zin van belang zijn voor het participeren in een gecomputeriseerde maatschappij.	computers and an education on doing mathematics with a computer system. (But G tends to forget the latter.)
Voor het rekenen komen we dan op zaken als het herkennen van rekenkundige problemen, het vertalen van dergelijke problemen in rekenopdrachten voor een computer, het begrijpen van die bewerkingen en het interpreteren en evalueren van antwoorden. Het gaat hier grofweg om toepassen, begrijpen en globaal rekenen.	• This is what the training on the use of the calculator has been about. • But beware: G claims to introduce the new magic phrase "global arithmetic". A term before 2000 was "computer savvy". • Computer algebra is a game changer, and indeed doesn't get sufficient attention. Why don't you explicitly say this ?
Begrijpen en toepassen behoren tot gangbare rekendoelen. Maar een keuze voor globaal rekenen leidt tot een grondige verandering van de leerstof. Om globaal te kunnen rekenen moet je beschikken over netwerken van getalrelaties en flexibel om kunnen gaan met eigenschappen van rekenoperaties.	G's misrepresentation is: • This supposedly novel concept of "global arithmetic" isn't included in current mathematics education – neither in "realistic mathematics education" (RME). • Supposedly the "computer" would be the new phenomenon causing this change.
Getalrelaties Bij het evalueren van berekeningen is het voldoende om globaal te kunnen bepalen wat het antwoord ongeveer moet zijn. Om een eenvoudig voorbeeld te geven: bij een opgave als 4 x 27 kan dit betekenen dat een leerling bedenkt dat het antwoord ruim 100 (4 x 25) is, een ander dat het minder is dan 120 (4 x 30). En weer een andere leerling kan bedenken dat er 108 (2 x 54) uitkomt. Idealiter zou het zo moeten zijn dat leerlingen díe getalrelaties gebruiken waar zij vertrouwd mee zijn.	• This "global arithmetic" turns out to be the competence to guesstimate what a model would generate. • It so happens that part of this is already included, both for handling a calculator, and in elementary school RME. • Unfortunately, in this RME, much of this guessing needlessly tends to replace proper accuracy when such accuracy should not be a problem. (The given examples can be calculated simply.)
Wanneer we willen dat leerlingen wat dit betreft goed beslagen ten ijs komen, dan moeten we investeren in het inoefenen van, en spelen met, getalrelaties die je veel kunt gebruiken.	What about learning the tables of addition and multiplication by heart ?
Voor vermenigvuldigen kunnen we bijvoorbeeld denken aan veelvouden van 25, 75, 125 en dergelijke, en het kunnen relateren van deze getallen aan kommagetallen, breuken en procenten. Het gaat uiteindelijk om netwerken van getalrelaties op basis waarvan leerlingen bijvoorbeeld kunnen bedenken dat 4 x 1,25 = 5, omdat 4 x 25 = 100, en dus 4 x 125 = 500, of, omdat 4 x 1,25 gelijk is aan 4 x 1¼.	• It is dubious whether it should be a learning goal to memorize what numbers are easy to calculate. • It is a learning goal however that students develop a sense of numbers and algebra, so that they can proceed to the next stage of quantification: handling unusual quantities and without getting lost.
Voor alle duidelijkheid: ik pleit niet voor allerlei regeltjes voor handig rekenen die de leerlingen zouden moeten leren toepassen. Wanneer de leerlingen beschikken over een netwerk van getalrelaties, kunnen ze deze getalrelaties als het ware opvatten als puzzelstukjes die ze zo kunnen combineren dat ze een antwoord vinden.	• If you do not plead for this, why give those misleading examples ? • If you agree to abolish RME and return to more traditional education, why not say so ?
Denk bijvoorbeeld aan het uitrekenen van 7 + 8. Wanneer we deze opgave voorleggen aan jonge kinderen die over een passend netwerk van getalrelaties beschikken, dan zullen de getallen 7 en 8 verschillende getalrelaties bij	• There is no criterion why 7 + 8 should associate with 7 + 3 = 10. Perhaps in the early grades of elementary school when the tables of addition haven't been learned yet, one might have a discussion on using

hen oproepen. Zoals bijvoorbeeld: 7 + 3 = 10, 7 + 7 = 14, 8 = 7 + 1, 7 = 5 + 2, en 8 = 5 + 3.	7 + 3 + 5. But such a pons asinorum better soon be replaced by learning the tables of addition and multiplication. • It is silly to compare such elementary outcomes with the understanding and skill of dealing with calculators and computers.
Die kunnen zij op verschillende manieren combineren tot een rekenzin die het goede antwoord oplevert. Zoals: 7 + 8 = 5 + 5 + 2 + 3 = 10 + 5, of: 7 + 8 = 7 + 7 + 1 = 14 +1, of: 7 + 8 = 7 + 3 + 5 = 10 + 5.	• Yes, back to first grade. • Still a confusion between calculator and computer algebra. • This is supposed to be the argument for a novel approach required for dealing the revolution of computer algebra ?
Reken-technisch gebruiken de leerlingen hier de 'associatieve' en de 'commutatieve' eigenschap. In het eerder genoemde voorbeeld van 5 x 25, gebruiken ze de 'distributieve' eigenschap: 5 x 25 = 4 x 25 + 1 x 25.	• Yes, for the development of a good sense of number and algebra: a discussion of the properties of association, commutation and distribution are advisable. • This would be in the current programme if RME hadn't created such a havoc.
Het gaat hier dus om het gebruiken van rekeneigenschappen en getalrelaties, en niet om het kiezen uit een repertoire van 'handige oplossingsstrategieën'.	This is a mispresentation. • Yes, distribution is implicit in this example. • The example above was introduced as coming from a "network of relations": which still is RME trying to allow students to develop number sense by trying. • Thus G has the implicit false suggestion that RME methods train students in understanding association, commutation and distribution. • It can be observed that Jan van de Craats has been criticising RME in this manner, mistaking Van Hiele networks of relations as handy solution algorithms. Thus Gravemeijer's article is partly a correct defence against this shallow critique by Van de Craats. Both are wrong however.
Dat vraagt een ander basisschoolprogramma dan rekenonderwijs dat opleidt tot het snel en routinematig oplossen van rekenopgaven.	The misrepresentation is: RME is defended (a) via the supposedly new phenomenon of the calculator or computer, (b) to answer to the pleas for more attention for algebra. (c) to downgrade traditional education on arithmetic as routine drilling.
De kracht van standaardprocedures is dat je geen rekening houdt met specifieke kenmerken van getallen: de procedure werkt altijd en je hoeft niet over de getallen na te denken. De keerzijde is dat je de hierboven beschreven getal- en rekenkennis niet ontwikkelt.	The misrepresentation is that traditional education would only be interested in routine drilling, and not in the development of other aspects, such as the development of sense of number and algebra, and transfer to applications. RME follows one particular road to get to sense for number and algebra, and doesn't see any alternative except drilling.
Een ander voordeel van standaardprocedures is dat ze efficiënt zijn in de kennis die ze gebruiken. Je hebt aan de basisautomatismen	• Traditional algorithms (like long division) are called "cyphering". This is denouncing writing as "lettering".

voor optellen en aftrekken en de tafels van vermenigvuldiging voldoende voor het uitvoeren van alle cijferalgoritmen. Maar ook hier is er weer een nadeel. Vermenigvuldigkennis die de gangbare tafels overstijgt – zoals veelvouden van 12, 15 en 25 – komen bij het cijferen niet aan de orde.	• It is suggested that traditional didactics would only be interested in drilling. • It is false that the tables of multiplication up to ten and the traditional algorithm would cause a problem for numbers like 12, 15 or 25. It is false that you would have to learn a table of multiplication for each number.
Onbenoemde getallen Netwerken van getalrelaties hebben nog een andere functie: ze spelen een belangrijke rol bij de overgang van benoemde naar onbenoemde getallen. Bij het rekenen met natuurlijke getallen gaat dit min of meer vanzelf. Getallen die eerst alleen nog betekenis hebben in combinatie met concrete hoeveelheden, zoals in 'vier knikkers', krijgen geleidelijk aan het karakter van objecten, die hun betekenis ontlenen aan een netwerken van getalrelaties. Het getal 4 wordt dan geassocieerd met 4 = 3 + 1, 4 = 2 + 2, 4 = 5 – 1, 4 = 8 : 2, enz.	• G refers to algebra as "unidentified numbers" but algebra is more than the possible interpretation by means of numbers. Algebra also concerns formal patterns. • This repeats the confusion as if "networks of connections between numbers" other than the tables of addition and multiplication would be a serious objective in the development of a sense of numbers and algebra. Perhaps this might be true for RME that doesn't rely on traditional algorithms and that forces students to try to find solutions. But then it should be presented as a disastrous consequence of RME and not as a valuable educational target.
Bij breuken ligt hier een probleem. Onderzoek van Bruin-Muurling laat zien, dat leerlingen in het PO vrijwel uitsluitend met breuken als benoemde getallen werken, terwijl in het VO wordt verondersteld dat de instromende leerlingen het niveau van de onbenoemde getallen al hebben bereikt. Voor een goede aansluiting moeten de breuken ook hun betekenis gaan ontlenen aan getalrelaties. Bij ¾ kan dat bijvoorbeeld zijn: ¾ = ¼ + ¼ + ¼ = 3 x ¼ , ¾ = 1 – ¼ , ¾ = ½ + ¼, of ¾ + ¾ = 1½, maar ook 3 : 4 = ¾, ¾ = 6/8 = 9/12 = 12/16 =..., en ¾ van 100 is 75, enz.	• Yes, elementary school in Holland has made it easy for itself by dropping algebraic understanding of fractions from the learning goals. They are happy when students can calculate sums, e.g. by using "tables of proportions", even when they have no insight in the formal form. • No, you cannot misrepresent and rephrase this as if this is related to such a "network". The objective to master algebra of division doesn't require the acquistion of such networks first.
Een goede basis Er is op dit moment veel aandacht voor de 'basisvaardigheden'. Daarbij wordt gemakshalve aangenomen dat basisvaardigheden voor rekenen onveranderlijk zijn. Maar wat zijn nu echt de vaardigheden die leerlingen van nu nodig hebben? Daar hoort het vlot en routinematig vermenigvuldigen van getallen van drie of vier cijfers volgens mij niet bij. Buiten het onderwijs gebruik je daar de rekenmachine voor. Maar ook voor de wiskunde in het voortgezet onderwijs heb je deze vaardigheid niet nodig.	This misrepresents the learning goals w.r.t. the fast and routine multiplication of figures with three or four digits. G suggests that people who advance this learning goal would negate the existence of the calculator, and would overstate the requirement for later algebra in highschool. Instead, the learning goal w.r.t. this activity is not in the result of the calculation, but is the command of the underlying operations, the understanding of the positional system, the use of memory for the various steps, the development of sense of number and algebra. It is a gross misrepresentation as if the learning goal for traditional didactics are that society would need that all people can do such calculations routinely. It is merely one of the useful *test formats* at the end of elementary school, for the stated purposes.
Wanneer je je afvraagt welke rekenkennis en vaardigheden een basis leggen voor algebra, dan kom je op andere zaken, zoals inzicht in	In these lines G reponds to criticism: that the RME method "try to find an answer" doesn't lay a foundation for algebra. He misrepresents this

de eigenschappen van rekenoperaties. Leerlingen moeten deze eigenschappen flexibel kunnen hanteren.	criticism, for he suggests that traditional didactics would think that routine calculation would provide such a foundation. The traditional didactics is to make students aware why the traditional algorithms work: which cause the awareness of the properties of association, commutation and distribution.
Zo zijn het vermenigvuldigen van tweetermen, ((a + b) x (c + d) = ac + ad + bc + bd), en de daarmee samenhangende merkwaardige producten, gebaseerd op het herhaald toepassen van de distributieve eigenschap. Dit flexibel gebruiken van eigenschappen van rekenoperaties is niet nieuw. Het heeft in Nederland een lange traditie in het zogeheten hoofdrekenen.	• G suggests that such algebraic understanding derives from calculation by heart (in one's head, without pen and paper), whence in his opinion this is better learned by RME, which would generate better number sense ("networks" other than tables of addition and multiplication). • But no, such algebraic understanding is based upon knowledge of the traditional algorithms and subsequent didactics on algebra itself (e.g. geometry of rectangles).
We zullen dus een centrale plaats moeten inruimen voor het ontwikkelen van netwerken van getalrelaties en het flexibel kunnen gebruiken van rekeneigenschappen. Alleen dan ontstaat er rekenonderwijs dat leerlingen zo goed mogelijk voorbereidt op de toekomst.	• G reformulates the objectives of RME but uses abstract terms so that readers do not see that RME is reformulated again. • In this way RME is linked to 21st century skills, to provde a "good future for students". Students who get this education will not understand what computers do. • G doesn't take the opportunity to say that RME has cause huge problems in mathematics education in Holland.
Tekst Koeno Gravemeijer, emeritus hoogleraar Eindhoven School of Education. Meer artikelen over reken- en wiskundeonderwijs voor de 21ste eeuw zijn te vinden op www.rekenenwisk21.nl	Gravemeijer has been a pillar of RME and wrote his 1994 thesis with supervisor Adri Treffers, another pillar of RME. Which RME fails. Why doesn't he openly say so ?

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Colignatus is the science name of Thomas Cool, econometrician and teacher of mathematics

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