An explanation for Wigner's "Unreasonable effectiveness of mathematics in the natural sciences"

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January 9 2015

Scientists can be fans of magic but they would decline it within their professional work. Yet there is this curious proposition by Eugene Wigner (1902-1995) of some "Unreasonable effectiveness of mathematics in the natural sciences" [1] that resorts to such magic - and which proposition finds mention without outright rejection by other writers, notably by Davis & Hersh on the mathematical experience [2], and recently in the book review by Burgess [3] on a book by Hacking. The latter triggers this response.

Wigner [1] states: "The first point is that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it." I disagree. The world is a wonder since we know so little about it but that is no reason to call it irrational. Yes, be amazed, but please do not turn this into magic. There is a very good explanation for the phenomenon indicated by Wigner.

My suggestion is that there is nothing "unreasonable" about the effectiveness of mathematics. When we regard mathematics as abstracting from the world, then the root lies in the world, and then it should not be surprising that the result may apply to some phenomena in that world. There is neither need for some Platonic view in which concepts "exist" as "ideas" in some magical realm outside of physics, for we are merely speaking about abstraction. Just to be sure: abstraction is defined as *leaving out other aspects.* Abstraction is nothing special but the mere ability of the brain to select some aspects of some mental model and drop (most) other aspects of it. That mental model will relate to empirical phenomena or sensations that the brain experiences.

I am neither mathematician nor physicist, but as econometrician I have some experience in the empirical sciences since 1982 and I got another degree in Leiden 2008 as teacher of mathematics. Let me point to the theory by Pierre van Hiele about the levels of insight in understanding mathematics. [4] It appears to be commonly thought that Van Hiele would see those levels only applicable to geometry but he presented them as a general theory of knowledge. [5] The Van Hiele theory explains that students operating at one level cannot imagine what it is at the other level. This also holds for the students at the highest level who can no longer imagine what they were struggling with in the past. This theory also partly explains why teaching math is rather incomprehendible to research mathematicians. The Van Hiele theory appears to be very relevant for the re-engineering of mathematics education. [6]

While insight is relevant for education, for the present discussion it is more enlightening to regard the levels as levels of abstraction. Application to Wigner's view generates "levels of unreasonableness" (in reversed order): which conveys the message that one should look at the issue in the proper perspective (bottom-up rather than top-down). When Wigner states "The complex numbers provide a particularly striking example for the foregoing. Certainly, nothing in our experience suggests the introduction of these quantities." then he presumes a paradise of simplicity of only one level, and he neglects the process in teaching, starting from the perception of a two dimensional plane and concluding with the formal development.

I also suppose that the evaluation of the relation between mathematics and physics should not be confused by interference by other topics of discussion. A supporter of Wigner might hold that this merely shifts the frontier of the "miracle", but my suggestion is that the answer has been given by abstraction, and that the following are really different topics.

(1) W.r.t. empirical modeling also involving human agents, we have different approaches: determinism, chance, volition. There is no experiment that will allow us to determine what is the right approach. We are forced to be pragmatic from the perspective of the objectives of the research. [7]
(2) Human views may be confused by chance events with potentially hidden determinism. That someone wins the lottery twice might seem absurd but given the number of lotteries held around the world it becomes more understandable. Wigner's description of physicists struggling with their subject and mathematics leaves out all failures and potentially hidden determinism.
(3) We are in need for a biological theory of the mind - with good definitions for the mind in relation to the empirical brain. Generally mathematics is seen as mental activity, with the formula's on the blackboard only as a record for communication. A suggestion [8] is that the mind may be defined as working with abstractions in general, created by processes in the brain. Mathematical abstractions (deserving that name) are merely those perfected by tradition and professional development. A re-engineering of mathematics may be required if we want that studies on the brain are to be useful for math education. For example 2½ is supposed to be two-and-a-half but reads as two-times-a-half and thus better be coded as 2 + ½. See [6] again.

I am no physicist and thus cannot experience the wonder that Wigner apparently experiences in his examples. I only have a highschool understanding of $E = mc^2$, and e.g. when the abstract notion of space is already defined as Euclidean space then I cannot phantom why physicists think that they are free to redefine space (or what this would mean) merely to get rid of measurement errors. [6] Yet I presume that some aspects are the same in all empirical sciences and naturally in mathematics. Supporters of Wigner will agree anyhow that "unreasonable" likely isn't a physical concept. Thus there should be scope for agreement.

PM. Wigner [1] refers to Galileo (1564-1642) doing an experiment on gravity with two objects dropped from the tower of Pisa. Viviani locates this event in 1589. The experiment was done before by Simon Stevin (1548-1620) on a tower in Delft around 1586. But is seems not to be in doubt that Galileo developed his gravity laws (*De Motu*). [9]

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References

[1] EUGENE WIGNER, The unreasonable effectiveness of mathematics in the natural sciences, *Communications in Pure and Applied Mathematics* **13** (1960), 1–14.

[2] PHILIP J. DAVIS AND REUBEN HERSH, *The Mathematical Experience*, Pelican 1983

[3] JOHN P. BURGESS, Book Review: Ian Hacking "Why Is There Philosophy of Mathematics at All?", *Notices of the American Mathematical Society,* (2014) 1345-1348

[4] PIERRE M. VAN HIELE, *Structure and Insight: A Theory of Mathematics Education*, Academic Press, 1986

[5] THOMAS COLIGNATUS, *Pierre van Hiele and David Tall: Getting the facts right*, mimeo, http://arxiv.org/abs/1408.1930

[6] THOMAS COLIGNATUS, *Conquest of the Plane. Using The Economics Pack Applications of Mathematica for a didactic primer on Analytic Geometry and Calculus,* Thomas Cool Consultancy & Econometrics, 2011

[7] THOMAS COLIGNATUS, *A Logic of Exceptions. Using The Economics Pack Applications of Mathematica for Elementary Logic,* Thomas Cool Consultancy & Econometrics, 2nd edition 2011

[8] THOMAS COLIGNATUS*, Education of mathematics and brain research,* mimeo, website 2011

[9] JONA LENDERING, *Review of Jozef Devreese en Guido Vanden Berghe (2011), "'Wonder en is gheen wonder'. De geniale wereld van Simon Stevin. 1548-1620",* http://mainzerbeobachter.com/2014/12/29/simon-stevin with a comment by Mark Nieuweboer (MNb) (An English edition of the book is*"'Magic Is No Magic': The Wonderful World of Simon Stevin"*, Wit Pr 2008 available at amazon)