Comments on the NAW Referee Report of June 11 2015

W.r.t. the article *"A condition by Paul of Venice (1369-1429) solves Russell's Paradox, blocks Cantor's Diagonal Argument, and provides a challenge to ZFC",*
versions May 20 and June 4 2015. Page numbers w.r.t. June 12.

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June 15 2015

There is an important difference between the May 20 and June 4 versions: the latter contains theorems, lemmata and corollaries: notably lemma 3.2 (page 12) that the Cantorian view implies the possibility of the Pauline view.

- While originally the May 20 had been submitted, it was very kind of NAW and the referee to also accept the June 4, when it was reported to the editors.
- However, curiously, the referee missed the theorems, lemmata and corollaries. Perhaps his or her mind had already been made up, and the kindness to consider the update did not materialize into a real consideration.
- Also, the view of the referee based upon the May 20 version appears to be biased.
- I have taken the liberty to slightly edit the June 4 version to what is now the June 12 version. This change was based upon my own continued thought on enlightening the issue, and not upon the referee report.
- This does not change anything materially about this refereeing process. There is only the idea to provide readers with the best argumentation. Thus, look here: http://thomascool.eu/Papers/ALOE/2014-11-14-Paul-of-Venice.pdf (old link, new file). If required, the version management can be found at: http://vixra.org/abs/1412.0235.
- I have decided that a full rewite is better. The theorems, lemmata en corollaries of June 4 improve the argument but make the paper clutttered again, and showing that ZFC is inconsistent is something else than deproving Cantor's Theorem. Nevertheless, the NAW referee report requires an answer.

Unfortunately, the editors of NAW only provided a PDF. It is notoriously difficult to convert these to an editing environment. The best solution now is to make snapshots. Via a wider margin those remain readable.

(1) The referee remark on ZFC versus ZF versus Z is superfluous. What is remarked may all be true, but, Bas Edixhoven of the Leiden & Delft syllabus asserted that he only accepted ZFC: the referee better respects this choice rather than turn it into a seemingly critical remark.
(2) It is not quite true that the paper uses Cantor's Theorem so say something about ZFC.
The condition inspired by Paul of Venice is used to say something about Russell, Cantor and ZFC.

> **Overall impression**
>
> The author uses the commonly presented proof that a set and its power set never have the same cardinality as a vehicle to raise questions about the standard axiomatization of set theory: the axioms of Zermelo and Fraenkel, ZF — there is no application of the Axiom of Choice in the article, so the C is not needed; indeed, as there no instance of the Replacement scheme either we could be working with Zermelo's axioms only, that system is sometimes referred to as Z.

(3) The phrase "sounds dramatic" creats drama. This is a misrepresentation. This is not something like "Logicomix" but a scientific paper.
(4) Overall impression of this referee report: that the case would be weak, is not substantiated.

> He repeats a few times that the 'consistency condition' of Paul of Venice causes the collapse of Cantor's proof, if not of the theorem, if not of the system Z. This sounds dramatic were not for the fact that his case is weak, to say the least.

(5) Entities Phi and Psi and not sets ? Well, Psi is in ZFC and thus a set (Edixhoven, Hart). There is lemma 3.2 (page 12) that Psi implies Phi, so why would Phi not be a set ? The referee does not substantiate the hesitations.

The entities Φ and Ψ

Much of the argumentation revolves around the entities Φ and Ψ — I refrain for the moment from calling them sets. Especially Appendix E contains various questions and complaints about the perceived unwillingness

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In reality: Appendix E explains the logical choices involved.

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to allow the author his freedom to have B free in $\varphi[x]$. The problem is that in 3.2 the author rephrases

(7) This is a put-down: "first asserting the existence of B and then demanding that B not be free in phi[x]". (May 20, this is changed in June 4 to gamma[x], to avoid confusion with Phi).
This phrasing makes what the paper does seem inconsistent by itself. It is an obvious misrepresentation.
In reality: the "and then" should be "while".

to allow the author his freedom to have B free in $\varphi[x]$. The problem is that in 3.2 the author rephrases
Comprehension/Separation by first asserting the existence of B and then demanding that B not be free in $\varphi[x]$. In E3 (1) he asks why one would lift $\varphi[x]$ out of the Axiom and the quantifier.

(8) This is a put-down: The refereee suggests that the author does not know that the Axiom of Separation (SEP) is on occasion also called an Axiom Schema.
In reality: The term "afscheidingsschema" is in the quote from Hart (page 24). There is no quantifier for gamma so it is obvious too.

(9) This is a put-down: The author takes the Cantorian view, to argue as if the Pauline view is only a misunderstanding of the proper way to see SEP.
In reality: Precisely the distinction between the Cantorian and the Pauline view shows that the author is quite aware of the point.
The article however questions the soundness of the Cantorian view. Criticism should be allowed and not be rejected because it is criticism.
Moreover, the May 20 version did not contain lemma 3.2, which is now in the June 4 version (page 12) that shows that the Cantorian view implies the Pauline view.
The referee acknowledges the receipt of the June 4 version, but apparently did not consider it.

to allow the author his freedom to have B free in $\varphi[x]$. The problem is that in 3.2 the author rephrases Comprehension/Separation by first asserting the existence of B and then demanding that B not be free in $\varphi[x]$. In E3 (1) he asks why one would lift $\varphi[x]$ out of the Axiom and the quantifier.

The answer is that one does not. Separation is an *Axiom Schema*, one axiom for each well-formed formula. For each formula ϕ one lists its free variables, takes a variable not in that list, B say, and adopts the universal closure of

$$\exists B\,\forall u\,(u \in B \Leftrightarrow (u \in A \wedge \phi))$$

as an axiom. Note the order: first ϕ, then B, and then the instance of Separation; we *insert* ϕ into the Separation template.

A good explanation for this choice can be found on page 19 of Levy's *Basic Set Theory*: in order to avoid confusion between the free occurrences of B in ϕ and the explicit mentions in the instance of Separation.

What would happen if we would not do this and try to get Φ by applying the 'tolerant' version? In both versions of Separation in E.1 we would have B as a free variable in ϕ ('parameter' in Weisstein's formulation) and we would have to prepend a universal quantifier $\forall B$.

(10) This is a put-down. What now follows is a ridiculisation of what a Pauline reading would involve. The paper clearly explains that *B* falls under the existential quantifier, so it is absurd that the referee claims that there is need of an additional universal quantifier.

Thus the instance of Separation that would produce the author's Φ would use the formula $(x \notin f(x)$ && $x \in B)$ as its $\phi[x, f, B]$ to produce

$$\forall A \, \forall f \, \forall B \, \exists B \, \forall x \, (x \in B \Leftrightarrow (x \in A \wedge (x \notin f(x) \;\&\&\; x \in B)))$$

(Both B and f are parameters in $\phi[x, f, B]$.)

A problematic point here is the pair of quantifiers $\forall B \, \exists B$; what is lost is that $\forall B$ comes from the B in $\phi[x, f, B]$ and that $\exists B$ comes from the Separation template.

On purely logical ground one can delete the universal quantifier so that we end up with

$$\forall A \, \forall f \, \exists B \, \forall x \, (x \in B \Leftrightarrow (x \in A \wedge (x \notin f(x) \;\&\&\; x \in B)))$$

This exhibits the difference between Ψ and Φ, in informal terms: the former is the result of applying a recipe, the latter is the solution to an equation (B is on both sides of the $\Leftrightarrow$).

(11) This is a put-down. Via the ridiculisation route, the referee ends up with still the same formula as from the Pauline interpretation in the paper.
But, then, a spurious interpretation is given, as if the one would be a *recipe* and the other *a solution to an equation.*

Compare "x = 2" and "x = 4 - x". Some software programs like ALGOL use "x := 2" to denote that x is set to 2, while "x = 2" then would be the equation that must be solved for x. In itself this is an important distinction.

However, in the Axiom of Separation, also in the Cantorian interpretation, there is only the equivalence. Thus, there is no distinction between "recipe" and "equation".

The upshot is that the referee creates the suggestion as if there would not be freedom to handle the logical conditions involved.
In reality, the Pauline interpretation sticks to the logical environment and uses the available freedom to impose different conditions.
Apparently the latter is a major improvement w.r.t. the universe that the referee is familiar with, and one can be amazed that this improvement is not recognized for what it is.

(12)The following is a rather perverse put-down. The referee recognizes that the June 4 version extends on the May 20 version with a discussion on the singleton.
Table 2 on page 5 of the June 4 & 12 version shows exactly, and in compact manner, what the referee now takes a half page to state: That Psi has a single solution while Phi may have multiple solutions.
Thus, the referee suggests that the author has not been aware of this.

In reality, the referee should have referred to Table 2, first row.

Subsequently, the referee claims that there should be a unique solution to Phi. There is nothing in the Axiom of Separation that imposes uniqueness. There is only "There is a B" which does not preclude multiple solutions.

Yes, the function *f* should be a function and not a correspondence. But the *B* need not be unique.

Finally, the referee misses that lemma 3.2 (page 2) shows that the Cantorian reading implies the possibility of the Pauline reading. Thus, the criticism directed to the author is actually a criticism directed to the referee.

The singleton case

The above was written based on the May 20 version. In the mean time I received the June 4 version with an analysis of the case where A is a singleton set and the symbols Ψ and Φ rather than Φ' and Φ, so I edited the previous section accordingly.

The singleton case exhibits a further weakness of the author's approach. Let $A = \{\alpha\}$. There are exactly two maps from A to $\mathcal{P}(A)$: f_1 and f_2, where $f_1(\alpha) = \emptyset$ and $f_2(\alpha) = A$. We can easily calculate

$$\Psi_1 = \{x \in A : x \notin f_1(x)\} = A$$

and

$$\Psi_2 = \{x \in A : x \notin f_2(x)\} = \emptyset$$

In either case we have $\Phi_i \neq f_i(\alpha)$.

In the 'Pauline' case we must solve the equation

$$(x \in B \Leftrightarrow (x \in A \wedge (x \notin f_i(x) \wedge x \in B))) \qquad (i)$$

once for $i = 1$ and once for $i = 2$ (the author assures us that one & can be used here, and I assume & is his version of $\wedge$) to get Φ_1 and Φ_2.

It turns out that equation (1) has two solutions because both

$$\alpha \in \emptyset \Leftrightarrow (\alpha \in A \wedge (\alpha \notin f_1(\alpha) \wedge \alpha \in \emptyset)))$$

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$$\alpha \in A \Leftrightarrow (\alpha \in A \wedge (\alpha \notin f_1(\alpha) \wedge \alpha \in A)))$$

have truth value 'TRUE', therefore the function f_1 does not have a unique Φ_1 associated to it.

Equation (2) does have a unique solution: the empty set.

The author claims that he can *define* his Φ but from a definition one would expect a unique outcome; the singleton case already shows that this need not happen.

In general it appears that every subset of the set Ψ can serve as a Φ. This is the principal difference between the Ψ and Φ constructs: the former produces a unique set, the latter allows for many outcomes.

In sum:

- The referee does not consider the analysis in full.
- The referee shows serious bias.
- The referee does not recognise the new questions and new results.
- The referee imposes arbitrary conditions that are no part of set theory (like uniqueness).
- The referee employs misrepresentations and put-downs to ridiculise the paper.

Hence this is a hostile referee, and this is not a proper peer review.
See http://en.wikipedia.org/wiki/Hostile_witness

Below is the "NAW referee report" in full.

Overall impression

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